Fall 2003
Math 152
COMMON EXAM 2
Test Form B

<u>PRINT</u>: Last Name: ______________________________ **First Name:** ______________________

Signature: ____ **ID:** _________________________

Instructor's Name: ____ **Section #** _____________

Instructor use only.

Multiple choice	
Q11	
Q12	
Q13	
Q14	
Q15	
Total	

<u>INSTRUCTIONS</u>

1. In **Part I** (Problems 1–10), mark the correct choice on your ScanTron form using a #2 pencil. *For your own records, also record your choices on your exam!* The ScanTrons will NOT be returned. Write your name and the color of your test on the ScanTron.
2. In **Part II** (Problems 11–15), write all solutions in the space provided. You may use the back of any page for scratch work, but all work to be graded must be shown in the space provided. CLEARLY INDICATE YOUR FINAL ANSWERS.
3. Turn off all electronic devices including cell phones.

PART I: MULTIPLE-CHOICE PROBLEMS

Each problem is worth 5 points: *NO* partial credit will be given. Calculators may *NOT* be used on this part.

1. Which of these integrals represents the arc length of $y = x^3$ from $x = 0$ to $x = 1$?

   a) $\int_0^1 \sqrt{1+x^6}\,dx$ b) $\int_0^1 \sqrt{1+3x^2}\,dx$ **c)** $\int_0^1 \sqrt{1+9x^4}\,dx$

   d) $\int_0^1 2\pi x^3\sqrt{1+9x^4}\,dx$ e) $\int_0^1 \sqrt{1+x^3}\,dx$

$$y' = 3x^2$$
$$1 + y'^2 = 1 + 9x^4$$

2. The error bound for the trapezoid rule is $E_T \le \dfrac{K(b-a)^3}{12n^2}$ where $K = \max\limits_{a\le x\le b} |f''(x)|$. If the trapezoid rule is applied to $\int_1^5 \sqrt{x}\,dx$, what is the *smallest* value of n which guarantees an error of at most $\dfrac{1}{1200}$?

   a) 60 b) 20 c) 30 **d)** 40 e) 50

$$f' = \frac{1}{2}x^{-1/2} f'' = -\frac{1}{4}x^{-3/2} \quad |f''_{\max}| = \frac{1}{4}$$
$$\frac{4^2}{12n^2} \le \frac{1}{1200} \qquad 40^2 \le n^2 \quad n \ge 40$$

3. By comparing the functions $\dfrac{1}{1+x^3}$ and $\dfrac{1}{x^3}$, what conclusion can be drawn about $\int_1^\infty \dfrac{1}{1+x^3}dx$?

   a) No conclusion is possible **b)** It converges c) It does not converge
   d) Its value is 1/2 e) Its value is 1

4. Does $\displaystyle\int_0^1 \frac{1+x}{\sqrt{x}} dx$ converge?

a) YES   b) NO

5. If y is the solution of

$$y' = \frac{2x^5}{y^2}, \qquad y(1) = 1,$$

then the value of y at $x = 2$ is

a) 4   b) 0   c) 1   d) 2   e) 3

$$\int y^2 \, dy = \int 2x^5 \, dx$$

$$y^3/3 = \frac{2x^6}{6} + c \qquad y^3 = x^6 + 3c, \quad c = 0$$

$$y = x^2$$

6. Evaluate $\displaystyle\int_1^\infty xe^{-x} \, dx.$

a) does not converge   **b)** $\frac{2}{e}$ c) $\frac{1}{e}$ d) 0   e) e

$$\big[-xe^{-x} - e^{-x} \big]_1^\infty \frac{2}{e}$$

7. The integrating factor for

$$y' - \frac{2y}{x} = \sin x, \qquad x > 0$$

is

a) $e^{-\cos x}$ **b)** $\frac{1}{x^2}$ c) x^2 d) $2\ln x$ e) $-2\ln x$

$$\int -\frac{2}{x}dx = -2\ln x \quad e^{\ln(x^{-2})} = x^{-2}$$

8. Find the moment about the y-axis (NOTE: not the x-axis) of the region bounded by $y = 4x^2$, the x-axis and the vertical lines $x = 1$, $x = 2$. (density = 1)

a) 7 b) $\frac{28}{3}$ **c)** 15 d) $\frac{248}{5}$ e) 20

$$\int_1^2 x4x^2\,dx = 4[x^4]_1^2 = 16 - 1 = 15$$

9. When the curve $y = e^x$ from $x = 0$ to $x = 1$ is rotated about the y-axis, which integral represents the surface area?

a) $\int_0^1 2\pi\sqrt{1+e^{2x}}\,dx$ b) $\int_0^1 2\pi e^x\sqrt{1+e^{2x}}\,dx$ c) $\int_0^1 2\pi x\sqrt{1+e^x}\,dx$

d) $\int_0^1 2\pi e^x\sqrt{1+e^x}\,dx$ **e)** $\int_0^1 2\pi x\sqrt{1+e^{2x}}\,dx$

$$2\pi \times \sqrt{1+e^{2x}}$$

10. The correct form of patial fractions for $\frac{x^2+2x-1}{x^3(x^2+1)}$ is

a) $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x^2+1}$ b) $\frac{A}{x^3} + \frac{B}{x^2+1}$ c) $\frac{A}{x^3} + \frac{Bx+c}{x^2+1}$

d) $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Ax+B}{x^2+1}$ **e)** $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Dx+E}{x^2+1}$

PART II. WORKOUT PROBLEMS

Each problem is worth 10pts. Calculators are NOT allowed on this part. Partial credit will be given.

11. A tank contains 100 gal. of pure water. Brine containing 2 lb. of salt per gallon is pumped in at 4 gal/min and the perfectly mixed solution is pumped out at the same rate. Set up and solve a differential equation for the amount $y(t)$ of salt in the tank at time t minutes.

$$y' = 8 - \frac{y}{100}4$$
$$y' + y/25 = 8$$
$$e^{t/25}y' + \frac{1}{25}e^{t/25}y = 8e^{t/25}$$
$$e^{t/25}y = 200e^{t/25} - 200$$
$$y = 200(1 - e^{-t/25})$$

12. Find the centroid of the region enclosed by the parabolas $y = 2x^2$ and $y = 3 - x^2$ (use any symmetry that you find).

$$\text{Area} = \int_{-1}^{1} 3 - 3x^2\, dx = [3x - x^3]_{-1}^{1} = 2 - (-2) = \underline{4}$$

$$M_x = \int_{-1}^{1} (3 - x^2 - 2x^2)\left(\frac{2x^2 + 3 - x^2}{2}\right) dx$$
$$= \int_{-1}^{1} \frac{(3 - 3x^2)(3 + x^2)}{2} dx = \int_{-1}^{1} \frac{9 - 6x^2 - 3x^4}{2}\, dx$$
$$= \int_{0}^{1} 9 - 6x^2 - 3x^4\, dx = 9 - 2 - 3/5$$
$$= 7 - 3/5 = 32/5$$

$$\bar{y} = \frac{M_x}{A} = 8/\mathbf{5}$$

13. Find $\displaystyle\int \frac{3x^2-x}{(x-1)(1+x^2)}dx.$

$$\begin{aligned}
\frac{3x^2-x}{(x-1)(1+x^2)} &= \frac{A}{x-1}+\frac{Bx+C}{1+x^2}\\
3x^2-x &= A(1+x^2)+(Bx+C)(x-1)\\
\underline{x=1} \quad 2 &= 2A, \quad \underline{A=1}\\
\underline{x^2|} \quad 3 &= A+B, \quad \underline{B=2}\\
\underline{x|} \quad -1 &= -B+C \quad C=1\\
\int \frac{1}{x+1}+\frac{2x}{1+x^2}+\frac{1}{1+x^2}dx &= \ln|x-1|+\ln(1+x^2)+\tan^{-1}x+C
\end{aligned}$$

14. Find the surface area generated by rotating the curve

$$x = t^2/2, \quad y = 3t, \quad 0 \le t \le 4,$$

about the x-axis.

$$\begin{aligned}
ds^2 &= dx^2+dy^2 = t^2\,dt^2+9\,dt^2\\
ds &= \sqrt{t^2+9}\,dt\\
\int &= \int_0^4 2\pi 3t\sqrt{t^2+9}\,dt\\
u &= t^2+9\\
du &= 2t\,dt
\end{aligned}$$

$$\begin{aligned}
\int_9^{25} 3\pi\sqrt{u}\,du &= [2\pi u^{3/2}]_9^{25}\\
&= 2\pi(125-27)\\
&= 2\pi 98 = \underline{196\pi}
\end{aligned}$$

15. Set up, but DO NOT EVALUATE, the integrals that give the total arc length of the curves surrounding the region enclosed by $y = x^2$ and $y = x^3$.

$$\int_0^1 \sqrt{1+4x^2}\,dx + \int_0^1 \sqrt{1+9x^4}\,dx.$$