

Fall 2005
Math 152
COMMON EXAM 1
Test Form A

PRINT: Last Name: _____ First Name: _____

Signature: _____ ID: _____

Instructor's Name: _____ Section # _____

Instructor use only.

Multiple choice	
Q11	
Q12	
Q13	
Q14	
Total	

INSTRUCTIONS

1. In **Part 1** (Problems 1–10), mark the correct choice on your ScanTron form using a #2 pencil. *For your own records, also record your choices on your exam!* The ScanTrons will be collected after 1 hour; they will NOT be returned.
2. In **Part 2** (Problems 11–14), write all solutions in the space provided. All work to be graded must be shown in the space provided. **CLEARLY INDICATE YOUR FINAL ANSWERS.**

Part I: Multiple Choice

Each problem is worth 5 points. No partial credit will be given. Calculators may not be used on this part.

1. Which integral is the area enclosed by $y = 1 - x^2$ and $y = x^2 + x$?

(a) $\int_{-\frac{1}{4}}^1 \left(-\frac{1}{2} + \frac{1}{2}\sqrt{1+4y} + \sqrt{1-y}\right) dy$ (b) $2\pi \int_{-1}^{\frac{1}{2}} x(1-2x^2-x) dx$
(c) $\int_{-1}^{\frac{1}{2}} (1-2x^2-x) dx$ (d) $\pi \int_{-1}^{\frac{1}{2}} [(1-x^2)^2 - (x^2+x)^2] dx$
(e) $\int_{-\frac{1}{4}}^1 \sqrt{1+4y} dy$

2. Consider the function $f(x) = 3x^2 - 2x$ over the interval $-1 \leq x \leq 1$. Find *all* numbers c in this interval such that $f(c)$ equals the average value of f .

(a) $c = -\frac{1}{3}$ (b) $c = 0$ (c) $c = 0, 1$ (d) $c = -\frac{1}{3}, 1$ (e) $c = 1$

3. Calculate $\int \sin^2 x \cos^2 x dx$.

(a) $\frac{1}{4} \sin^4 x + C$ (b) $\frac{1}{8}x - \frac{1}{32} \sin 4x + C$
(c) $\frac{1}{12} \sin^3 2x + C$ (d) $\frac{1}{3} \sin^3 x - \frac{1}{5} \sin^5 x + C$ (e) $\frac{1}{2}x - \frac{1}{4} \sin^2 2x + C$

4. Calculate $\int_0^3 \frac{x \, dx}{\sqrt{x+1}}$.

(a) 0 (b) $\frac{8}{3}$ (c) 12 (d) $\frac{3}{2} - \ln 2$ (e) 9

5. Compute $\int_1^2 x \ln x \, dx$

(a) $2 \ln 2 - \frac{3}{4}$ (b) $2 \ln 2$ (c) $\ln 2$ (d) $4 \ln 2 - \frac{7}{2}$ (e) $\frac{1}{2}$

6. Calculate $f'(x)$ if $f(x) = \int_0^{\frac{1}{x}} \sqrt{t^3 + 1} \, dt$.

(a) $\frac{3x^2}{2\sqrt{x^3+1}}$ (b) $-\frac{\sqrt{x^3+1}}{x^2}$ (c) $\frac{1}{\sqrt{x^3+1}}$
 (d) $-\frac{1}{x^2} \sqrt{\frac{1}{x^3} + 1}$ (e) $\frac{3/x^2}{2\sqrt{1/x^3+1}}$

7. Consider the region between $y = f(x)$ and $y = g(x)$ over $0 \leq x \leq 2$, where $f(x) \geq g(x) \geq 0$. If we revolve this region about the line $y = -1$, which integral is the volume of the generated solid?

(a) $\pi \int_{-1}^2 [f(x)^2 - g(x)^2] \, dx$ (b) $2\pi \int_{-1}^2 x(f(x) - g(x)) \, dx$
 (c) $\pi \int_0^2 [(f(x) + 1)^2 - (g(x) + 1)^2] \, dx$ (d) $2\pi \int_0^2 (x + 1)(f(x) - g(x)) \, dx$
 (e) $\int_0^2 (f(x) - g(x)) \, dx$

8. Calculate $\int \frac{dx}{x(x-1)(x-2)}$.

- (a) $\ln|x| \ln|x-1| \ln|x-2| + C$
- (b) $\ln|x| + \ln|x-1| + \ln|x-2| + C$
- (c) $\frac{1}{2} \ln|x| - \ln|x-1| + \frac{1}{2} \ln|x-2| + C$
- (d) $-\frac{1}{2x^2} + \frac{1}{(x-1)^2} - \frac{1}{2(x-2)^2} + C$
- (e) $\frac{1}{2}(x-1)(x-2) - x(x-2) + \frac{1}{2}x(x-1) + C$

9. Suppose the force needed to displace an object depends on the position of the object according to the function

$$F(x) = 3 \sin(\pi x).$$

If the object is displaced from $x = 0$ to $x = 1$, calculate the work that is done.

- (a) 6
- (b) $\frac{6}{\pi}$
- (c) 6π
- (d) $\frac{\pi}{6}$
- (e) $\frac{1}{6}$

10. Calculate $\int_2^4 \frac{x}{x^2+3} dx$.

- (a) $\tan^{-1}\left(\frac{4}{\sqrt{3}}\right) - \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$
- (b) $\frac{4}{19} - \frac{2}{7}$
- (c) $\frac{1}{2} \ln 19 - \frac{1}{2} \ln 7$
- (d) $\frac{6}{25} - \frac{3}{13}$
- (e) $\frac{1}{49} - \frac{13}{361}$

Part II: Work-Out Problems

Calculators are permitted. Show your work. Partial credit is possible. An answer supported only by your calculator is not acceptable.

11. Calculate the following integrals. (5 points each)

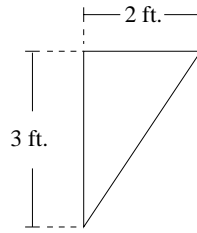
(a) $\int_0^1 \frac{dx}{x^2 + 2x + 2}$

(b) $\int \tan^3 x \sec x \, dx$

(c) $\int x^2 e^{2x} \, dx$

(d) $\int \sqrt{4 - x^2} \, dx$

12. Consider a trough full of water which is 8 ft. long and whose cross-section is the right triangle



Calculate the work needed to pump all of the water over the top. Assume that the density of water is 62.5 lb./ft.^3 (10 points)

13. Consider the region enclosed by $y = x^2 - x + 1$ and $y = 2x - 1$. Find the volume of the solid generated by revolving this region about the y -axis. (10 points)

14. Consider the solid with the disk $x^2 + y^2 \leq 9$ in the xy -plane as its base. If every cross-section perpendicular to the x -axis is an equilateral triangle, what is the volume of the solid? (10 points)