

Maplets for Calculus – Tutoring without the Tutor¹

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Abstract: *Finding eye-catching computer algebra-based resources is easy. The challenge is to find effective ways to use them --- within the classroom and beyond. Resources that merely solve the problem for the students or show them the solution do not engender true learning. The students need to be thinking through the solution process on their own. This paper focuses on one specific set of instruction applets, Maplets for Calculus, and how they can be used as effective educational tools for both students and instructors.*

Maplets for Calculus (M4C) is a collection of 94 Maple applets illustrating most of the topics in single variable calculus. (See <http://m4c.math.sc.edu/> or <http://m4c.math.tamu.edu/>.) Each maplet is designed to be useable in several different settings: in a laboratory for student exploration, in the classroom for demonstrations, or by students seeking additional practice with immediate feedback. The maplets use a combination of symbolic, graphic (2D or 3D, sometimes with animation), numeric and verbal devices to investigate problems. Problems are either algorithmically generated or explicitly entered by the student or instructor. The maplets require the user to work through the problem step-by-step. Correct answers to earlier steps are required before moving on to later steps. Hints are available and when errors are detected, instructive feedback is provided. All of this enables students to use M4C as "a tutor without the tutor."

Students find the algorithmically-generated problems very helpful for practicing new techniques and for building confidence as they prepare for a quiz or exam. Immediate feedback and infinite patience are two features students find useful. Students find the hints and step-by-step checking of their responses as effective as a private tutor. For example, Figure 1 shows the Integration by Substitution maplet in which the user has omitted the differential in the substitution.

Instructors have multiple opportunities to use the Maplets for Calculus to enhance their courses. The ability to launch a Maplet with a specific example makes it convenient to use a Maplet for demonstrations and examples in a lecture. Plus, the extensive use of graphics – 2D, 3D, and animation – add to the impact that the Maplets can have in the classroom. Examples include the maplets: Epsilon Delta Definition of a Limit (Figure 2), Derivatives of Inverse Functions (Figure 3), Volume by Slicing (Figure 4) and Area as an Antiderivative (Figure 5). Others will find it more appropriate to develop a project or a lab that utilizes a Maplet in a guided-discovery exercise. For example, the Epsilon Delta Definition of a Limit maplet provides a graphical and interactive environment for exploring the ϵ - δ definition of a limit (Figure 2). Instructors can also have their students drill on difficult topics such as Indefinite or Definite Integration Techniques and Series Convergence Tests (Figure 6 — Figure 9).

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A Table of Contents and sample videos of the Maplets for Calculus may be seen at <http://m4c.math.tamu.edu/>. An expanded and updated version of Maplets for Calculus was completed in January 2008. The remainder of this paper shows snapshots taken from several different maplets in the M4C collection.

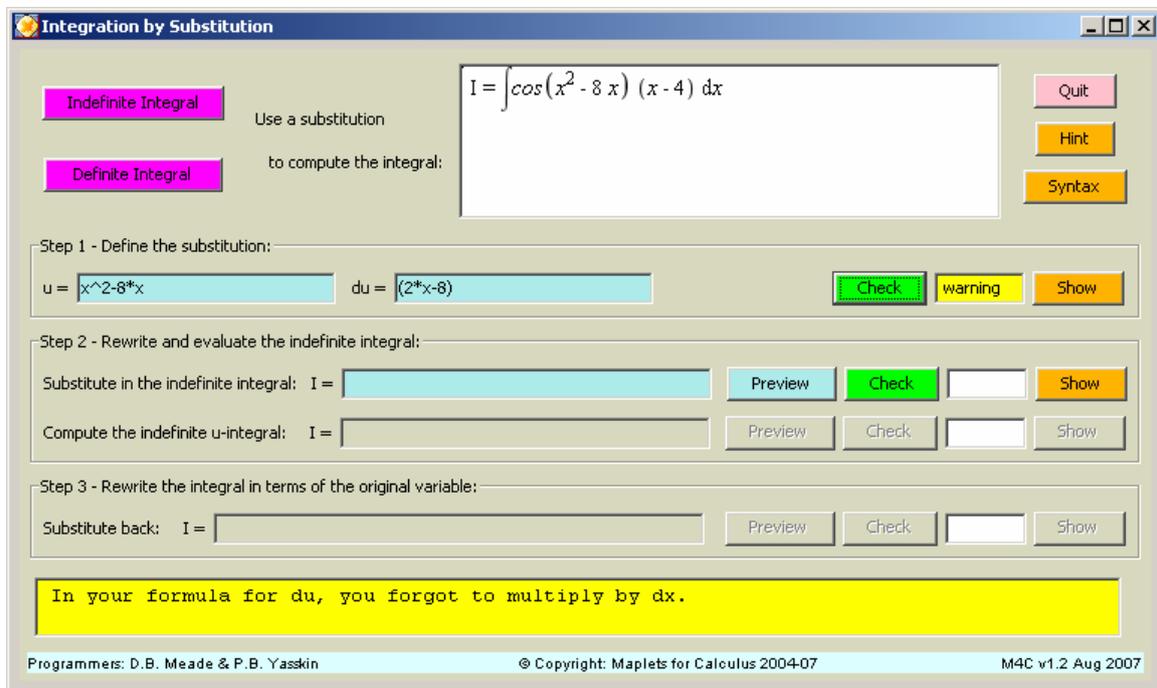


Figure 1: Constructive feedback in the [Integration by Substitution](#) maplet. Note that the user has omitted the differential (dx) in their response for du .

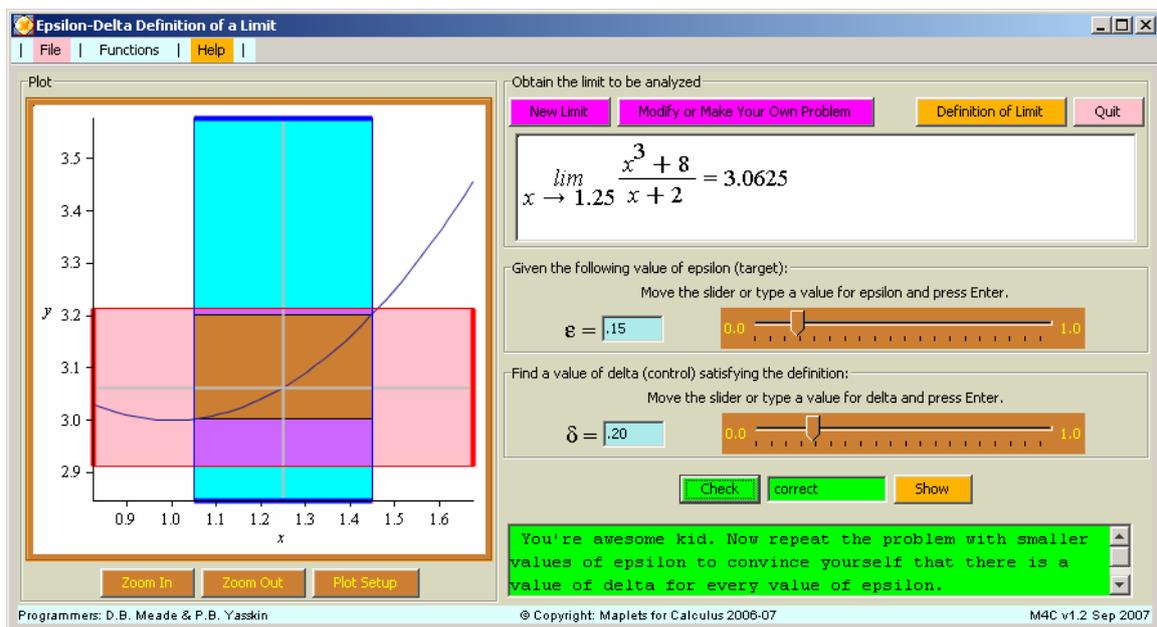


Figure 2: Visualizing the ϵ - δ definition of the limit with the [Epsilon Delta Definition of a Limit](#) maplet. Users can control the values of ϵ and δ using the sliders, or by entering a value in the corresponding input field.

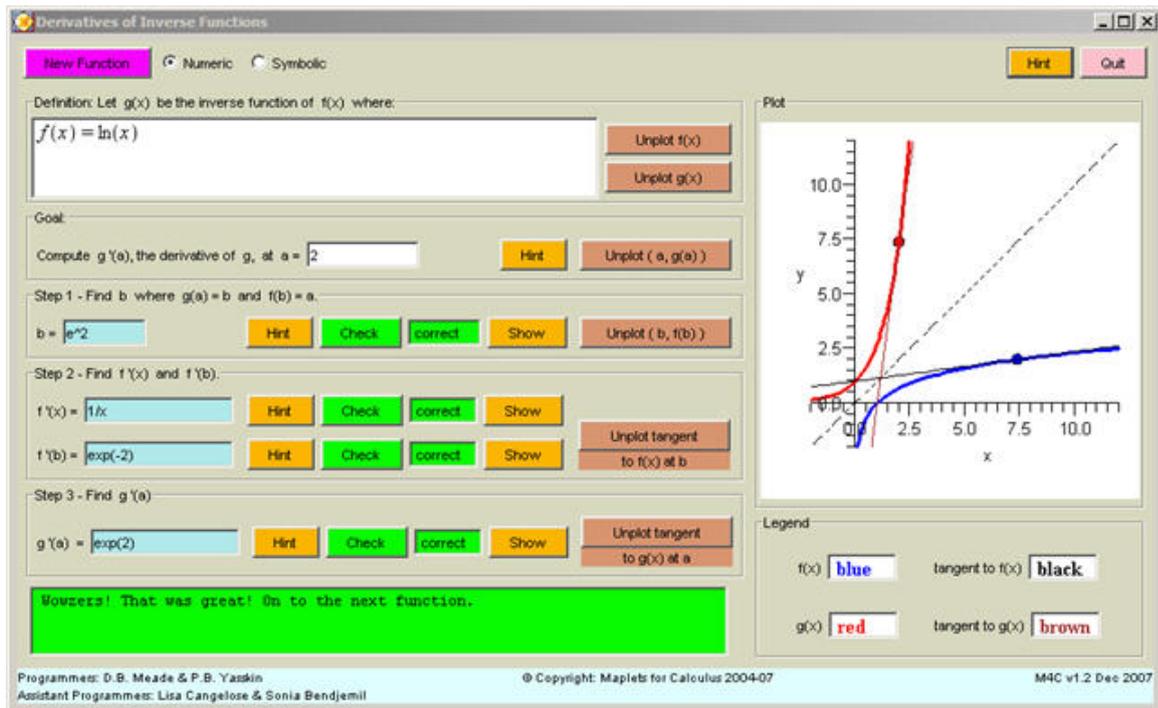


Figure 3: The [Derivatives of Inverse Functions](#) maplelet, showing a function and its inverse along with tangent lines at mirror image points.

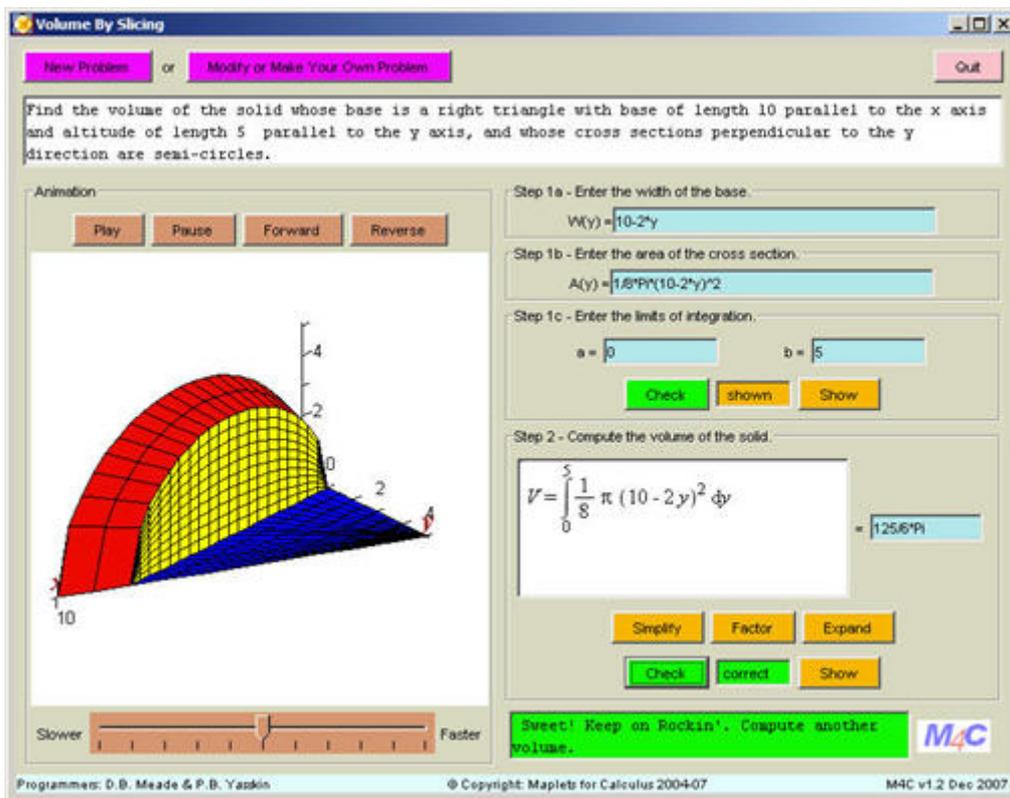


Figure 4: The [Volume By Slicing](#) maplelet, showing one frame of the 3D animation and the emphasis on the setup of the definite integral.

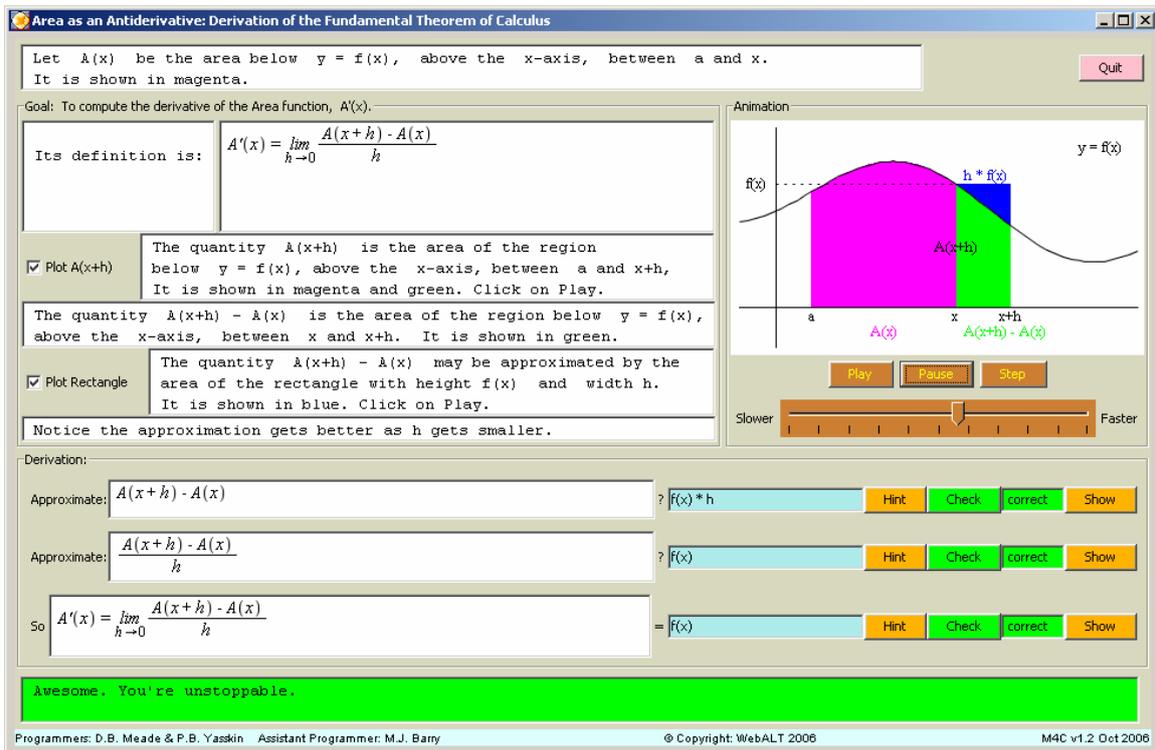


Figure 5: In the [Area as an Antiderivative: Derivation of the Fundamental Theorem of Calculus](#) maplet an animation helps students understand why the FTC is true.

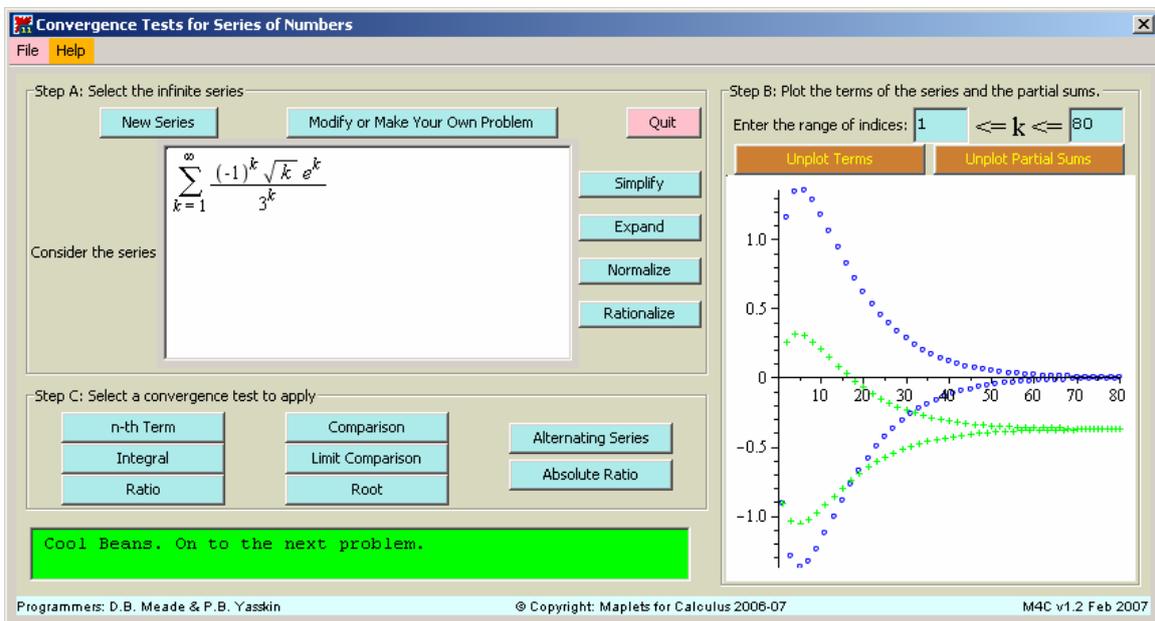


Figure 6: The [Convergence Tests for Series of Numbers](#) maplet. When users select a convergence test to apply, the test appears in a separate popup window. (See the next 3 figures.) Note the graphical display of the terms (blue) and the partial sums (green).

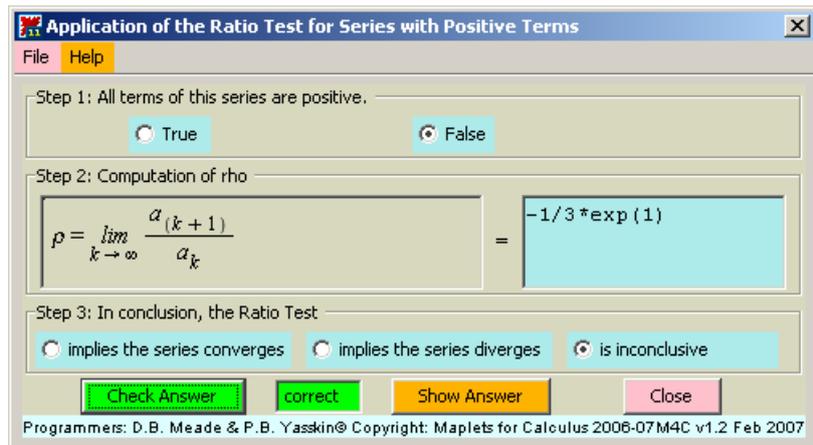


Figure 7: The popup window for the Ratio Test. This test is inconclusive because the terms of this series are not all positive.

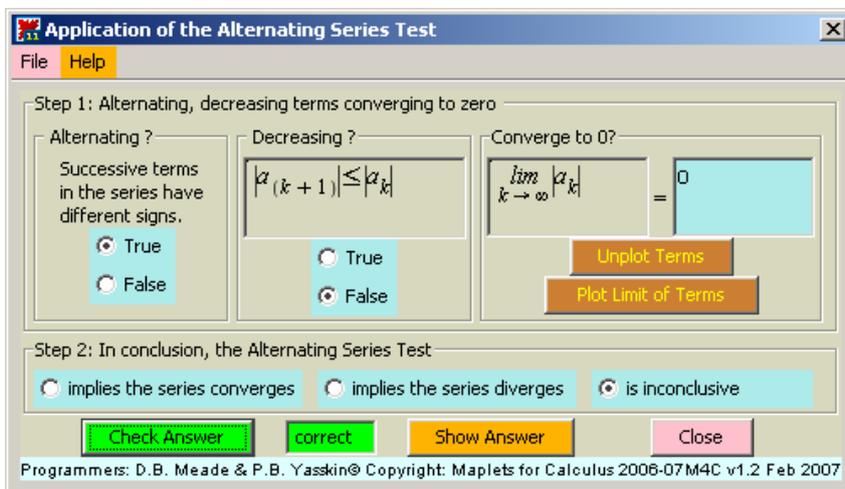


Figure 8: The popup window for the Alternating Series Test. This test cannot be applied because the sequence of terms increases (in absolute value) for the first few terms.

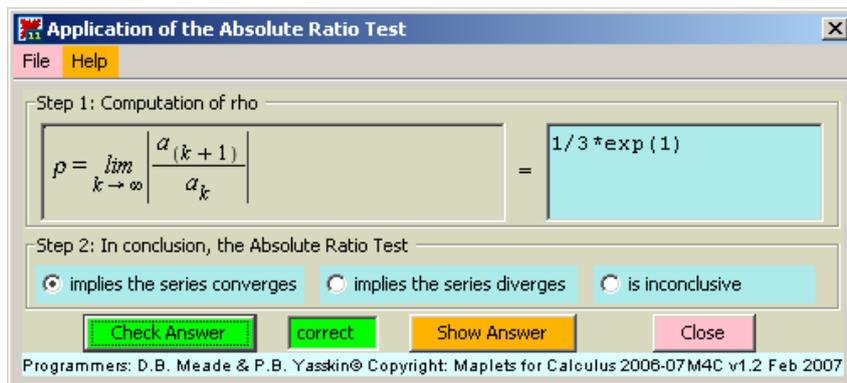


Figure 9: The popup window for the Absolute Ratio Test. This test can be used to show the convergence of this series.

Appendix. List of Maplets for Calculus

The 94 maplets in Maplets for Calculus are divided into the following eight categories:

1. Pre-Calculus (22 maplets)
 - graphical properties of functions (translations, reflections, domain, range, asymptotes)
 - absolute value, quadratic, exponential, logarithmic and trigonometric functions
 - review of trig
2. Limits (8 maplets)
 - conceptual understanding of limits and continuity, one-sided and two sided
 - ϵ - δ definition of the limit (graphical and formal proof)
 - intermediate value theorem and bisection method
3. Derivatives (24 maplets)
 - basic principles: secant lines and tangent lines
 - practice with the terminology of derivatives and with making graphical connections between properties of a first or second derivative and the original function
 - drill and practice on differentiation techniques (chain rule, implicit differentiation, logarithmic differentiation, inverse functions)
 - applications (linear and quadratic approximations, related rates, motion, l'Hospital's rule, max/min problems)
 - antiderivatives and an introduction to the Fundamental Theorems of Calculus
4. Integrals (28 maplets)
 - focused practice with specific integration techniques (substitution, integration by parts, trigonometric integrals, trigonometric substitution, partial fractions)
 - drills for definite and indefinite integration
 - geometrical applications (area of a region, volume by slicing, volume of revolution, arc length, surface area)
 - physical applications (average value, mass, center of mass, work, fluid force)
 - numeric integration (left and right Riemann sums, trapezoid and Simpson's rules)
5. Differential Equations (4 maplets)
 - drills for both separable and linear differential equations
 - direction fields
 - mixing problems
6. Sequences and Series (4 maplets)
 - general drills for limits of sequences and convergence of series
 - more focused practice with geometric and telescoping series
7. Curvilinear Coordinates (2 maplets)
 - identification of polar curves
8. Games (2 maplets)
 - Plotting
 - cryptograms