Science Course for High School Students
Problems from Engineering Mathematics
Tuesday, 20/Jan/2009 Art Belmonte, TAMUQ

Calculus 1 Problems

1. Find all horizontal and vertical asymptotes of $f(x) = \dfrac{x^2 - x - 6}{2x^2 + 2x - 24} + \dfrac{x-1}{x+1}$.
2. Compute $\lim_{x \to -\infty} \left(\sqrt{x^2 + 7x - 9} + (x+4) \right)$.
3. If position is given by $r(t) = t^2 - 4t + 1$, find the average velocity between times $t = 1$ and $t = 1.1$.
4. Calculate the slope of the tangent line to the curve $x^2 - y^2 = 3(1 + xy)$ at $(x, y) = (1, -1)$.
5. Compute the quadratic approximation to $f(x) = 4x + 2\cos x$ at $a = 0$.
6. Evaluate $\lim_{x \to 0^+} \left(x + e^{x/3} \right)^{3/x}$.
7. Find an equation of the tangent line to the curve $y = 2\left(x + e^{x-1} + \tan^{-1} x\right)$ at $x = 1$.
8. Cesium-137 and strontium-90 are two radioactive chemicals that were released at the Chernobyl nuclear reactor in April 1986. The half-life of cesium-137 is $h_1 = 30.22$ years, and that of strontium-90 is $h_2 = 28.8$ years. (Recall that the half-life of a substance is the amount of time it takes for a given amount to decay to one-half of its original amount.)
   (a) How much time must pass before the amount of cesium is equal to 1% of what was released?
   (b) Answer this question for strontium.
9. Compute $\displaystyle\int_1^4 \frac{3x+2}{\sqrt{3x^2 + 4x}}\, dx$.
10. A closed rectangular box with a square base is to be constructed.
    - The material for the sides and top lid costs \$0.25 per square foot.
    - The bottom costs \$0.50 per square foot.
    - Find the dimensions in feet of a box with volume 2 cubic feet that has minimum cost. Also state this cost.
11. Find a unit vector perpendicular to $\mathbf{v} = \begin{bmatrix} -4 & 7 \end{bmatrix}$.
12. Determine the work done by the force field $\mathbf{F} = 2\mathbf{i} - 3\mathbf{j}$ in moving a particle from $A(-1, 4)$ to $B(2, -5)$.

Calculus 2 Problems

1. Find the area between the curves $y = x^2 - 2x - 2$ and $y = x + 2$.
2. Compute the volume obtained by revolving the region in the first quadrant bounded by the lines $y = 0$, $x = 0$, $x = \pi$, and the curve $y = 2 + \sin 2x$, about the y-axis.
3. Find the average value of $f(x) = \cos^2 x + \sin 2x$ on the interval $\left[0, \frac{\pi}{4}\right]$.
4. Evaluate the indefinite integral $\int x^7 e^{x^2}\, dx$.
5. Compute $\displaystyle\int \frac{x^3}{\sqrt{36 - 4x^2}}\, dx$.
6. Evaluate the infinite sum $\displaystyle\sum_{n=4}^{\infty} \frac{24}{(n+4)(n+5)}$.
7. Calculate the arc length of the curve $y = \ln(\cos x)$, $0 \le x \le 1$.
8. Find the area of the surface obtained by rotating the curve $x = 3t - t^3$, $y = 3t^2$, $0 \le t \le 1$, about the x-axis.
9. Determine the center of mass of the flat plate bounded by the curves $y = x^2$ and $y = 12 - x$. The plate's density is constant.
10. Determine the convergence of the series $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left(\frac{n}{n+1}\right)^{n^2}$.
11. Find the Maclaurin series for e^{-x^5}.
12. Find the third degree Taylor polynomial for $f(x) = \dfrac{1}{\sqrt[4]{1+x}}$ at $a = 0$.
13. Compute $\displaystyle\lim_{x \to 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5}$.
14. Show that the set of points $P(x, y, z)$ such that the distance from P to $A(-1, 5, 3)$ is twice the distance from P to $B(6, 2, -2)$ is a sphere. Find its center and radius.
15. Find the volume of the parallelepiped (sheared box) determined by the vectors $\mathbf{u} = \begin{bmatrix} 4 & -5 & 7 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 6 & -1 & -3 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 2 & 8 & -1 \end{bmatrix}$.

(Please turn the page over for problems from Calculus 3 and Differential Equations.)

Calculus 3 Problems

1. Find a parametric representation of the tangent line to the curve $x = 1 + 2\sqrt{t}$, $y = t^3 - t$, $z = t^3 + t$ at $(3, 0, 2)$.
2. Classify the critical points of
$$f(x, y) = x^3 - 8xy + 27y^3$$
as local maxima, local minima, or saddle points.
3. The base of a rectangular aquarium with volume 160 ft^3 is to be made of slate costing $\$5/\text{ft}^2$. The sides of glass will cost $\$1/\text{ft}^2$. The aquarium has no top. Find the dimensions of the aquarium that minimizes the cost of materials. Also give this minimum cost.
4. Find the area inside the cardioid $r = 8 + 8\sin\theta$ and above the line $y = 6$ (i.e., $r\sin\theta = 6$ or $r = 6/\sin\theta$).
5. Find the center of mass of the lamina (flat plate) in the first quadrant of the xy-plane bounded by the curve $y = 1 + \sqrt{x}$ and the line $y = \frac{1}{3}x + 1$. The variable density of the lamina is $\rho = xy$.
6. Find the mass of the quarter-circular wire in the lower half of the xy-plane that lies between the lines $y = x$ and $y = -x$ on the circle $x^2 + y^2 = 81$. The density of the wire is $\rho = -y$ (which is positive in the lower half of the xy-plane since y is negative there).
7. Find the work done by the force field $\mathbf{w} = \left[e^z, \ln(z+1), \tan^{-1} z\right]$ on a particle that moves along the curve $\mathbf{g}(t) = \left[t^3, t^2, t\right]$, $0 \le t \le 1$.
8. Find an equation of the tangent plane to the curve $z = \sqrt{x + e^{xy}}$ at $(x, y) = (3, 0)$.
9. Find the flux $\iint_S \mathbf{w} \cdot d\mathbf{S}$ of the vector field $\mathbf{w} = [x, y, 6]$ outward across the two-piece surface that consists of the paraboloid $z = x^2 + y^2$, $0 \le z \le 16$, and the circular disk $x^2 + y^2 = 16$, $z = 16$. Do it directly or use a theorem.
10. Calculate the integral $\int_0^4 \int_{\sqrt{y}}^2 \frac{y^2 e^{x^2}}{x}\, dx\, dy$ by reversing the order of integration. (First draw a picture of the region of integration.)
11. Find the volume of the apple whose boundary is the surface $\rho = \phi$.

(Please turn the page over for problems from Calculus 1 and Calculus 2.)

Differential Equations Problems

1. Solve the initial value problem
$$\frac{dy}{dx} = \frac{y^2 + 1}{y}, y(0) = 2.$$
2. Solve the initial value problem
$$\left(4x^3 + 3y\right) dx + \left(3x + 4y^3\right) dy = 0, \quad y(1) = 1.$$
3. Find a general solution to the differential equation
$$\left(1 + x^3\right) y' = 3x^2 y + x^2 + x^5.$$
4. A tank initially contains 100 gal of water in which is dissolved 2 lb of salt. A salt-water solution containing 1 lb of salt for every 4 gallons enters the tank at a rate of 5 gal per minute. The well-mixed solution leaves the tank at the same rate.
   (a) Find the amount y of salt in the tank at time t.
   (b) Graph y for $0 \le t \le 20$.
   (c) When is there 7 lb of salt in the tank?
   (d) What is the limiting amount of salt in the tank?
5. Soup initially at 205° F cools to 193° F after one minute.
   (a) Find the temperature of the soup at time t.
   (b) Graph this temperature for $0 \le t \le 10$.
   (c) When is the temperature of the soup 150° F, at which time it is ready to eat?
6. Find a general solution to the second-order homogeneous differential equation $y'' + 6y' + 9y = 0$.
7. Find a solution to the initial value problem
$$y'' + 2y' + 10y = 0, \quad y(0) = 2.$$
8. Find a general solution to the second-order differential equation nonhomogeneous differential equation $y'' - y' = 3t + 4$.
9. Solve the initial value problem $y'' + 16y = 8\cos 4t$, $y(0) = 0$, $y'(0) = 0$, then graph the solution for $0 \le t \le 4\pi$. What phenomenon does this graph exhibit?
10. Solve the initial value problem
$$4y'' + 1024y = 50\cos 15t, \quad y(0) = 0, \quad y'(0) = 0,$$
then graph the solution for $0 \le t \le 6\pi$.
What phenomenon does this graph exhibit?