

**High School Science Program / Math  
Exam 1 (Calculus portion): Solutions  
Sun, 01/Feb ©2009 Art Belmonte**

1. Find the tangent line to  $y = x^3 - 6x^2$  at  $x = 3$ .  
Write it in slope-intercept form,  $y = mx + b$ .

- $y_0 = f(x_0) = f(3) = 27 - 54 = -27$
- $f'(x) = 3x^2 - 12x$
- $m = f'(x_0) = f'(3) = 27 - 36 = -9$
- $y - y_0 = m(x - x_0)$  whence  $y + 27 = -9(x - 3)$
- $y = -9x$

2. Find the area between  $y = x^2 - 4x + 6$  and  $y = 2x + 6$ .  
Lengths are in cm. Include units of area in your final answer.

- Intersections:

$$\begin{aligned}x^2 - 4x + 6 &= 2x + 6 \\x^2 - 6x &= 0 \\x(x - 6) &= 0 \\x &= 0, 6\end{aligned}$$

- At  $x = 1$ , parabola's  $y$ -value is 3, whereas that of the line is 8. Hence the line lies above the parabola.
- Area:

$$\begin{aligned}A &= \int_0^6 2x + 6 - (x^2 - 4x + 6) dx \\&= \int_0^6 6x - x^2 dx \\&= \left(3x^2 - \frac{1}{3}x^3\right) \Big|_0^6 \\&= \left(3(6)^2 - 2(6)^2\right) - (0 - 0) \\&= 6^2 = 36 \text{ cm}^2\end{aligned}$$