Fall 2007 Math 151 Quiz 2B Thu, 13/Sep/2007

Name: **UIN:** **Circle section: 804 805 806**

For all quizzes, no calculators. The only thing on your desk should be your PENCIL. Circle your choices AND mark them on your QuizzStrip Scantron #815-E. The 4 problems are each worth 5 points; total: 20 points. *Please write legibly!*

1. Evaluate the limit $\lim_{x\to 6^+} \dfrac{\sqrt[4]{x^2-36}}{x+6}$.

   (a) 0 (b) ∞ (c) $\sqrt[4]{12}$ (d) It does not exist.

   - (a) Via direct substitution, the limit is $\dfrac{0}{12} = 0$.

2. The limit $\lim_{x\to 3} \dfrac{x^2+x-a}{x-3}$ equals 7 when:

   (a) $a = -12$ (b) $a = 12$ (c) Never (d) $a = 0$

   - (b) With $a = 12$, we have $\lim_{x\to 3} \dfrac{x^2+x-12}{x-3} = \lim_{x\to 3} \dfrac{(x-3)(x+4)}{(x-3)} = \lim_{x\to 3} (x+4) = 7$.

3. The function $f(x) = \begin{cases} \dfrac{4-x}{2-\sqrt{x}} & \text{if } x > 4 \\ 6 & \text{if } x = 4 \\ \dfrac{x^2-2x-8}{x-4} & \text{if } x < 4 \end{cases}$

   (a) is right-continuous at $x = 4$, but not left-continuous there.

   (b) is left-continuous at $x = 4$, but not right-continuous there.

   (c) is continuous at $x = 4$.

   (d) neither left-continuous nor right-continuous at $x = 4$.

   - (b) Now $\lim_{x\to 4^-} \dfrac{x^2-2x-8}{x-4} = \lim_{x\to 4^-} \dfrac{(x+2)(x-4)}{(x-4)} = \lim_{x\to 4^-} (x+2) = 6 = f(4)$, whereas $\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} \dfrac{4-x}{2-\sqrt{x}} = \lim_{x\to 4^+} \dfrac{\left(2-\sqrt{x}\right)\left(2+\sqrt{x}\right)}{\left(2-\sqrt{x}\right)} = \lim_{x\to 4^+} \left(2+\sqrt{x}\right) = 4 \neq 6 = f(4)$. Therefore, f is left-continuous at $x = 4$, but not right-continuous there.

4. Find the limit $\lim_{x\to\infty} \dfrac{3-\sqrt{x}+\sqrt[4]{x}}{1+\sqrt{x}+\sqrt[3]{x}}$.

   (a) -1 (b) 3 (c) 0 (d) ∞

   - (a) We have

$$\lim_{x\to\infty} \frac{3-x^{1/2}+x^{1/4}}{1+x^{1/2}+x^{1/3}} = \lim_{x\to\infty} \frac{\dfrac{3}{x^{1/2}}-1+\dfrac{1}{x^{1/4}}}{\dfrac{1}{x^{1/2}}+1+\dfrac{1}{x^{1/6}}} = \frac{0-1+0}{0+1+0} = -1.$$