Fall 2007 Math 151 Quiz 3A Solutions Tue, 25/Sep/2007

Name: **UIN:** **Circle section: 819 820 821**

For all quizzes, no calculators. The only thing on your desk should be your PENCIL. Circle your choices AND mark them on your QuizzStrip Scantron #815-E. The 4 problems are each worth 5 points; total: 20 points. *Please write legibly!*

1. Find an equation of the tangent line to the curve $y = \dfrac{1}{1+x^2}$ at the point $\left(2, \frac{1}{5}\right)$.

   (a) $y = \frac{4}{25}x - \frac{3}{25}$ (b) $y = \frac{1}{25}x + \frac{3}{25}$ (c) $y = -\frac{4}{25}x + \frac{13}{25}$ (d) $y = -\frac{1}{25}x + \frac{2}{25}$

   - The slope of the tangent line is $m = y'(2) = \left.\dfrac{\left(1+x^2\right)(0) - (1)(2x)}{\left(1+x^2\right)^2}\right|_{x=2} = -\dfrac{4}{25}$.
   - (c) An equation of the tangent line is $y - \frac{1}{5} = -\frac{4}{25}(x-2)$ or $y = -\frac{4}{25}x + \frac{13}{25}$.

2. Which of these piecewise functions are differentiable at $x = 0$?

$$f(x) = \begin{cases} x \sin\left(\dfrac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases} \qquad g(x) = \begin{cases} x^3 \sin\left(\dfrac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

   (a) f (b) g (c) f and g (d) neither

   - Since $\lim\limits_{x\to 0} \dfrac{f(x) - f(0)}{x - 0} = \lim\limits_{x\to 0} \dfrac{x \sin\left(\frac{1}{x}\right) - 0}{x - 0} = \lim\limits_{x\to 0} \sin\left(\frac{1}{x}\right)$ does not exist [$\sin\left(\frac{1}{x}\right)$ diverges by oscillation as $x \to 0$], we see that f is not differentiable at $x = 0$.
   - Since $\lim\limits_{x\to 0} \dfrac{g(x) - g(0)}{x - 0} = \lim\limits_{x\to 0} \dfrac{x^3 \sin\left(\frac{1}{x}\right) - 0}{x - 0} = \lim\limits_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$, we see that g is differentiable at $x = 0$.
   - (b) Hence the correct answer is (b).

3. Evaluate the expression $\dfrac{\left(x^2+4x-21\right)(x-3)' - (x-3)\left(x^2+4x-21\right)'}{\left(x^2+4x-21\right)^2}$. Here the prime symbol ($'$) signifies differentiation with respect to x. [HINT: Think "reverse engineering."]

   (a) 1 (b) $\dfrac{-1}{x^2+14x+49}$ (c) $\dfrac{1}{x+7}$ (d) $\dfrac{-1}{x^2-6x+9}$

   - (b) If you're clever, you recognize the expression as an application of the quotient rule. (Look at its *form*.) It represents

$$\frac{d}{dx}\left(\frac{x-3}{x^2+4x-21}\right) = \frac{d}{dx}\left(\frac{(x-3)}{(x-3)(x+7)}\right) = \frac{d}{dx}\left(\frac{1}{x+7}\right) = \frac{(x+7)(0) - (1)(1)}{(x+7)^2} = \frac{-1}{x^2+14x+49}.$$

   - [Brute force and algebraic simplification also works. . .]

4. Suppose that $f(5) = 1$, $f'(5) = 6$, $g(5) = -3$, and $g'(5) = 2$. Let $h = fg$. Compute $h'(5)$.

   (a) -3 (b) 12 (c) -16 (d) 9

   - (c) The product rule yields

$$\begin{aligned} h' &= f'g + fg' \\ h'(5) &= f'(5)\,g(5) + f(5)\,g'(5) \\ h'(5) &= (6)(-3) + (1)(2) = -16. \end{aligned}$$