Fall 2007 Math 151 Quiz 3B Solutions Tue, 25/Sep/2007

Name: **UIN:** **Circle section: 804 805 806**

For all quizzes, no calculators. The only thing on your desk should be your PENCIL. Circle your choices AND mark them on your QuizzStrip Scantron #815-E. The 4 problems are each worth 5 points; total: 20 points. *Please write legibly!*

1. Find an equation of the tangent line to the curve $y = \dfrac{2}{x-2}$ when $x = 3$.

   (a) $y = -x + 5$ (b) $y = 2x - 4$ (c) $y = \frac{1}{4}x + \frac{5}{4}$ (d) $y = -2x + 8$

   - The slope of the tangent line is $m = y'(3) = \left.\dfrac{(x-2)(0) - (2)(1)}{(x-2)^2}\right|_{x=3} = -2.$
   - (d) An equation of the tangent line is $y - 2 = -2(x - 3)$ or $y = -2x + 8$.

2. Differentiate $f(x) = x\sqrt{x} + \dfrac{1}{x^2\sqrt{x}}$.

   (a) $x^2\sqrt{x} + \dfrac{5}{x\sqrt{x}}$ (b) $\dfrac{\sqrt{x}}{2} + \dfrac{2}{x^2\sqrt{x}}$ (c) $\sqrt{x} + \dfrac{1}{x^2}$ (d) $\dfrac{3\sqrt{x}}{2} - \dfrac{5}{2x^3\sqrt{x}}$

   - (d) Rewrite the function as $f(x) = x^{3/2} + x^{-5/2}$. Then $f'(x) = \frac{3}{2}x^{1/2} - \frac{5}{2}x^{-7/2}$, which is equivalent to the last choice.

3. Which of these piecewise functions are differentiable at $x = 0$?

$$f(x) = \begin{cases} x^{1/3}\sin\left(\dfrac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases} \qquad g(x) = \begin{cases} x^4\sin\left(\dfrac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

   (a) f (b) g (c) f and g (d) neither

   - Since $\displaystyle\lim_{x\to 0}\frac{f(x) - f(0)}{x - 0} = \lim_{x\to 0}\frac{x^{1/3}\sin\left(\frac{1}{x}\right) - 0}{x - 0} = \lim_{x\to 0}\frac{\sin\left(\frac{1}{x}\right)}{x^{2/3}}$ does not exist because $\dfrac{\sin\left(\frac{1}{x}\right)}{x^{2/3}}$ oscillates with increasing amplitude as $x \to 0$. We see that f is not differentiable at $x = 0$.
   - Since $\displaystyle\lim_{x\to 0}\frac{g(x) - g(0)}{x - 0} = \lim_{x\to 0}\frac{x^4\sin\left(\frac{1}{x}\right) - 0}{x - 0} = \lim_{x\to 0} x^3\sin\left(\frac{1}{x}\right) = 0$, we see that g is differentiable at $x = 0$.
   - (b) Hence the second choice is the correct answer.

4. Suppose that $f(5) = -3$, $f'(5) = -9$, $g(5) = 4$, and $g'(5) = 2$. Let $h = fg$. Compute $h'(5)$.
   (a) -12 (b) -18 (c) -42 (d) 30

   - (c) The product rule yields

$$\begin{aligned} h' &= f'g + fg' \\ h'(5) &= f'(5)\,g(5) + f(5)\,g'(5) \\ h'(5) &= (-9)(4) + (-3)(2) = -42. \end{aligned}$$