Fall 2007 **Math 151** **Quiz 4A Solutions** **Tue, 09/Oct/2007**

Name: **UIN:** **Circle section: 819 820 821**

For all quizzes, no calculators. The only thing on your desk should be your PENCIL. Circle your choices AND mark them on your QuizzStrip Scantron #815-E. The 4 problems are each worth 5 points; total: 20 points. *Please write legibly!*

1. Let $f(x) = \dfrac{\sin x + \sec x + 1}{\cot x + 1}$. Find $f'\left(\dfrac{\pi}{4}\right)$.

   (a) $\dfrac{3\sqrt{2}+1}{2}$ (b) $\sqrt{2} + \frac{1}{2}$ (c) 1 (d) -1

   - Now $f'(x) = \dfrac{(\cot x + 1)(\cos x + \sec x \tan x) - (\sin x + \sec x + 1)\left(-\csc^2 x\right)}{(\cot x + 1)^2}$ via the Quotient Rule.
   - (a) Thus $f'\left(\dfrac{\pi}{4}\right) = \dfrac{(2)\left(\frac{\sqrt{2}}{2} + \sqrt{2}\right) - \left(\frac{\sqrt{2}}{2} + \sqrt{2} + 1\right)(-2)}{(4)} = \dfrac{3\sqrt{2} + 3\sqrt{2} + 2}{4} = \dfrac{6\sqrt{2}+2}{4} = \dfrac{3\sqrt{2}+1}{2}$.

2. Compute $g'\left(\dfrac{\pi}{4}\right)$ given $g(x) = \cos^2\left(\tan^2 x\right)$.

   (a) $4\cos 1 \sin 1$ (b) $-8\cos 1 \sin 1$ (c) $-4\cos 1 \sin 1$ (d) $8\cos 1 \sin 1$

   - We have $g'(x) = 2\cos\left(\tan^2 x\right) \cdot \left(-\sin\left(\tan^2 x\right)\right) \cdot 2\tan x \cdot \sec^2 x$ via recursive use of the Chain Rule.
   - (b) Hence $g'(\pi/4) = 2\cos 1 \cdot (-\sin 1) \cdot 2 \cdot 2 = -8\cos 1 \sin 1$.

3. Find the slope of the tangent line to the curve $y^5 + 3x^2y^2 + 5x^4 = 9$ at the point $(1, 1)$.

   (a) $-\dfrac{20}{11}$ (b) $-\dfrac{26}{5}$ (c) $-\dfrac{26}{11}$ (d) $-\dfrac{17}{11}$

   - (c) Implicit differentiation yields

   $$\begin{aligned} 5y\frac{dy}{dx} + 6xy^2 + 3x^2\left(2y\frac{dy}{dx}\right) + 20x^4 &= 0 \\ \left(5y + 6x^2y\right)\frac{dy}{dx} &= -\left(6xy^2 + 20x^4\right) \\ \frac{dy}{dx} &= -\frac{6xy^2 + 20x^4}{5y + 6x^2y} = -\frac{26}{11} \end{aligned}$$

   when $(x, y) = (1, 1)$.

4. Find a unit tangent vector to the parametric curve $\mathbf{r}(t) = [\csc t, \cot t]$ for $t = \pi/4$.

   (a) $\left[\dfrac{1}{2}, \sqrt{\dfrac{3}{4}}\right]$ (b) $\left[-\dfrac{1}{\sqrt{3}}, \sqrt{\dfrac{2}{3}}\right]$ (c) $\left[\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right]$ (d) $\left[-\dfrac{1}{\sqrt{3}}, -\sqrt{\dfrac{2}{3}}\right]$

   - Compute $\mathbf{r}'(t) = \left[-\csc t \cot t, -\csc^2 t\right]$.
   - A tangent vector is $\mathbf{v} = \mathbf{r}'(\pi/4) = \left[-\sqrt{2}, -2\right]$.
   - (d) Therefore a unit tangent vector is $\hat{\mathbf{v}} = \dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{\left[-\sqrt{2}, -2\right]}{\sqrt{2+4}} = \left[-\sqrt{\dfrac{2}{6}}, -\sqrt{\dfrac{4}{6}}\right] = \left[-\dfrac{1}{\sqrt{3}}, -\sqrt{\dfrac{2}{3}}\right]$.