

Name:

UIN:

Circle section: 819 820 821

For all quizzes, no calculators. The only thing on your desk should be your PENCIL. Circle your choices AND mark them on your QuizzStrip Scantron #815-E. The 4 problems are each worth 5 points; total: 20 points.

Please write legibly!

1. Find the point on the curve $x = t^2 + 1$, $y = t^3 - 3t$ where the tangent line is vertical.

- The tangent line to the curve will be vertical where $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \pm\infty$, colloquially speaking. More precisely, this can occur where $dx/dt = 0$ and $dy/dt \neq 0$ simultaneously.
- Now $dx/dt = 2t = 0$ implies $t = 0$. For this value of t we have $dy/dt = 3t^2 - 3 = -3 \neq 0$.
- (d) Accordingly, the tangent line is vertical for $t = 0$. Plugging this value into the parametric equations for x and y gives $(x, y) = (1, 0)$.

2. Let $f(x) = \sin x$. Find the sixth derivative of $f(x)$, then evaluate it at $x = \pi/3$.

- The first six derivatives of $f(x)$ are $\cos x$, $-\sin x$, $-\cos x$, $\sin x$, $\cos x$, $-\sin x$, respectively. (Note that the derivatives repeat in a cycle of length 4.)
- (c) Accordingly, the sixth derivative of $f(x)$ at $x = \pi/3$ is $-\sin(\pi/3) = -\sqrt{3}/2$.

3. Find the linear approximation of the function $f(x) = \sqrt{x+3}$ at $a = 1$, then use it to approximate $\sqrt{4.01}$.

- Write $f(x) = (x+3)^{1/2}$. Then $f'(x) = \frac{1}{2}(x+3)^{-1/2}$.
- Thus $f(1) = 2$ and $f'(1) = \frac{1}{4}$.
- Hence the linear approximation is $L(x) = f(1) + f'(1)(x-1) = 2 + \frac{1}{4}(x-1)$.
- (b) Finally, $\sqrt{4.01} = f(1.01) \approx L(1.01) = 2 + \frac{1}{4}(1.01-1) = 2.0025$.

4. A glider flying horizontally at an altitude of 1 mile and a velocity of $48\sqrt{2}$ miles per hour (mph) passes directly over a radar station. Determine the rate at which the distance between the radar station and the glider is changing when the glider is 3 miles away from the station (diagonally through the air).

- Let x be the distance along the ground from the radar station to the point directly below the glider.
- Let z be the distance between the radar station and the glider diagonally through the air.
- (b) By the Pythagorean Theorem we have $z^2 = x^2 + 1^2$. Thus

$$\begin{aligned} 2z \frac{dz}{dt} &= 2x \frac{dx}{dt} \\ \frac{dz}{dt} &= \frac{x}{z} \frac{dx}{dt} \\ \frac{dz}{dt} &= \frac{\sqrt{8}}{3} (48\sqrt{2}) = 64 \text{ mph.} \end{aligned}$$