Fall 2007 Math 151 Quiz 5B Solutions Tue, 16/Oct/2007

Name: **UIN:** **Circle section: 804 805 806**

For all quizzes, no calculators. The only thing on your desk should be your PENCIL. Circle your choices AND mark them on your QuizzStrip Scantron #815-E. The 4 problems are each worth 5 points; total: 20 points. *Please write legibly!*

1. Find the linear approximation of $f(x) = \sqrt{1+x}$ at $a = 0$.
   - Write $f(x) = (1+x)^{1/2}$. Then $f'(x) = \frac{1}{2}(1+x)^{-1/2}$.
   - Thus $f(0) = 1$ and $f'(0) = \frac{1}{2}$.
   - (c) Hence the linear approximation is $L(x) = f(0) + f'(0)(x-0) = 1 + \frac{1}{2}x$.
2. Given $y = f(x) = 3x^2 + \sin x$, compute the differential dy for $x = \pi$ and $dx = 2$.
   - We have $dy = f'(x)\, dx = (6x + \cos x)\, dx$.
   - (c) Plugging in $x = \pi$ and $dx = 2$ gives $dy = (6\pi - 1)(2) = 12\pi - 2$.
3. At what value of t among those listed is the tangent line to the curve $x = \cos t - \sin t$, $y = 1 + \cos t$ vertical?
   - The tangent line to the curve will be vertical where $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} =$ "$\pm\infty$", colloquially speaking. More precisely, this can occur where $dx/dt = 0$ and $dy/dt \neq 0$ simultaneously.
   - Now $dx/dt = -\sin t - \cos t = 0$ for $t = 3\pi/4$ (alone among the choices that were listed). For this value of t we have $dy/dt = -\sin t = -\sqrt{2}/2 \neq 0$.
   - (b) Accordingly, the tangent line is vertical for $t = 3\pi/4$.
4. A triangle has two fixed sides of lengths 6 m and 10 m. The angle θ between these sides is changing at a rate of 3 rad/s. Find the rate (in m^2/s) at which the area of the triangle is increasing when the angle between these sides is $\pi/3$.
   - Let b and h represent the length of the base and the height of the triangle respectively.
   - The area of the triangle is $A = \frac{1}{2}bh$, which for this problem is equivalent to $A = \frac{1}{2}(10)(6\sin\theta) = 30\sin\theta$.
   - (a) Differentiating with respect to t yields

     $$\frac{dA}{dt} = 30\cos\theta \cdot \frac{d\theta}{dt} = 30\left(\frac{1}{2}\right)(3) = 45\ \text{m}^2/\text{s}$$

     when $\theta = \pi/3$ and $d\theta/dt = 3$.