

Name:

UIN:

Circle section: 819 820 821

For all quizzes, no calculators. The only thing on your desk should be your PENCIL. Circle your choices AND mark them on your QuizzStrip Scantron #815-E. The 4 problems are each worth 5 points; total: 20 points.

Please write legibly!

1. Find the quadratic approximation of $f(x) = \cos x$ at $a = 0$, then use it to approximate $\cos(0.01)$.

- Now $f(0) = \cos 0 = 1$, $f'(0) = -\sin 0 = 0$, and $f''(0) = -\cos 0 = -1$.
- Hence the desired quadratic approximation is

$$Q(x) = f(0) + f'(0)(x-0) + \frac{f''(0)}{2}(x-0)^2$$

$$Q(x) = 1 - \frac{1}{2}x^2.$$

- (a) Therefore, $\cos(0.01) \approx Q(0.01) = 1 - \frac{1}{2}(0.01)^2 = 1 - 0.00005 = 0.99995$.

2. Let $f(x) = x^4 - 10$. Use Newton's Method with $x_1 = 2$ to approximate x_2 , the next approximation to the positive root of the equation $f(x) = 0$.

- (b) Let $g(x) = x - \frac{f(x)}{f'(x)} = x - \frac{x^4 - 10}{4x^3}$. Then $x_2 = g(2) = 2 - \frac{16 - 10}{32} = 2 - \frac{3}{16} = \frac{29}{16}$.

3. Obtain an equation of the tangent line to the curve $y = x^2e^{-x}$ at $(1, 1/e)$. Now, what is the value of y on the tangent line for $x = e/2$?

- Now $y' = 2xe^{-x} - x^2e^{-x}$, whence $y'(1) = \frac{2}{e} - \frac{1}{e} = \frac{1}{e}$.
- The point-slope formula yields $y - \frac{1}{e} = \frac{1}{e}(x - 1)$ or $y = \frac{x}{e}$ for the tangent line.
- (d) Hence on the tangent line at $x = e/2$, we have $y(e/2) = \frac{e/2}{e} = \frac{1}{2}$.

4. Let $g(x)$ be the inverse function $f^{-1}(x)$ of $f(x) = x^5 - x^3 + 2x$. Find $g'(2)$.

- By inspection, $f(1) = 2$, whence $g(2) = 1$, since g is the inverse of f .
- (c) Accordingly, $g'(2) = \frac{1}{f'(g(2))} = \frac{1}{f'(1)} = \frac{1}{(5x^4 - 3x^2 + 2)|_{x=1}} = \frac{1}{4}$.