Fall 2007 Math 151 Quiz 6B Solutions Tue, 23/Oct/2007

Name: **UIN:** **Circle section: 804 805 806**

For all quizzes, no calculators. The only thing on your desk should be your PENCIL. Circle your choices AND mark them on your QuizzStrip Scantron #815-E. The 4 problems are each worth 5 points; total: 20 points. *Please write legibly!*

1. Find the quadratic approximation of $f(x) = \sqrt{x+1}$ at $a = 0$.

   - With $f(x) = (x+1)^{1/2}$, we have $f'(x) = \frac{1}{2}(x+1)^{-1/2}$ and $f''(x) = -\frac{1}{4}(x+1)^{-3/2}$.
   - Thus $f(0) = 1$, $f'(0) = \frac{1}{2}$, and $f''(0) = -\frac{1}{4}$.
   - (c) Hence the desired quadratic approximation is

$$\begin{aligned} Q(x) &= f(0) + f'(0)(x-0) + \frac{f''(0)}{2}(x-0)^2 \\ Q(x) &= 1 + \tfrac{1}{2}x - \tfrac{1}{8}x^2. \end{aligned}$$

2. Let $f(x) = e^{x+1} - 10x$. Use Newton's Method with $x_1 = 0$ to approximate x_2, the next approximation to a root of the equation $f(x) = 0$.

   - (b) Let $g(x) = x - \dfrac{f(x)}{f'(x)} = x - \dfrac{e^{x+1} - 10x}{e^{x+1} - 10}$. Then $x_2 = g(0) = 0 - \dfrac{e-0}{e-10} = \dfrac{e}{10-e} \approx 0.37$.

3. Obtain an equation of the tangent line to the curve $f(x) = e^{-x}(1 + \sin x)$ at $x = \pi$.

   - First, $f(\pi) = e^{-\pi}$.
   - Now $f'(x) = -e^{-x}(1+\sin x) + e^{-x}\cos x$, whence $f'(\pi) = -2e^{-\pi}$.
   - (d) The point-slope formula yields $y - e^{-\pi} = -2e^{-\pi}(x-\pi)$ or $y = -2e^{-\pi}(x-\pi) + e^{-\pi}$ for the tangent line.

4. If the domain of $f(x) = \sin x$ is restricted to the interval $\left[0, \frac{1}{2}\pi\right]$, what is the derivative of $g(x)$, the inverse function $f^{-1}(x)$ of $f(x)$ at $x = 1/\sqrt{2}$?

   - Now $f(\pi/4) = 1/\sqrt{2}$, whence $g\left(1/\sqrt{2}\right) = \pi/4$, since g is the inverse of f.
   - (a) Accordingly, $g'\left(1/\sqrt{2}\right) = \dfrac{1}{f'\left(g\left(1/\sqrt{2}\right)\right)} = \dfrac{1}{f'(\pi/4)} = \dfrac{1}{\cos(\pi/4)} = \dfrac{1}{1/\sqrt{2}} = \sqrt{2}$.