Fall 2007 Math 151 Quiz 8B Solutions Tue, 13/Nov/2007

1. Find the value of x at which a local minimum of $f(x) = \frac{1}{5}x^5 + x^3 - 4x + 1$ occurs.

   - Solve $0 = f'(x) = x^4 + 3x^2 - 4 = (x^2 - 1)(x^2 + 4) = (x - 1)(x + 1)(x^2 + 4)$ to obtain critical values $x = \pm 1$.
   - (a) We see that the sign of f' changes from $-$ to $+$ only at $x = 1$, where a local minimum occurs.

2. What is the largest interval (among those given) on which $y = e^{-x^2/2}$ is concave up?

   - (b) Now $f'(x) = -xe^{-x^2/2}$. Thus $f''(x) = (x^2 - 1)e^{-x^2/2} > 0$ for $|x| > 1$. Hence f is concave upward on $(1, \infty)$ among the intervals given.

3. Consider the function $f(x) = 2x^3 + 9x^2 - 24x$. Find the values of x at which the absolute maximum and absolute minimum of f on $[0, 2]$ occur.

   - Solve $0 = f'(x) = 6x^2 + 18x - 24 = 6(x - 1)(x + 4)$ to obtain critical values $x = -4, 1$, only one of which is in the interval $[0, 2]$.
   - (d) Compute function values of f at the critical value $x = 1$ and the endpoints to locate where the absolute maximum and absolute minimum occur.

x	0	1	2
$f(x)$	0	-13	4

     We see that the max occurs at $x = 2$ and the min at $x = 1$.

4. You want to fence in an area along a straight stream that runs directly west to east. The two sides perpendicular to the stream require rust-proof fencing costing \$10 per foot while the side opposite the stream costs \$5 per foot. (There is no fencing along the stream itself.) What is the maximum area (in square feet) that can be enclosed if you have \$400 to spend?

   - Let x be the length of the fence parallel to the stream and y the length of each piece of fence perpendicular to the stream.
   - The cost constraint requires $5x + 10(2y) = 400$, whence $y = \frac{400-5x}{20} = 20 - \frac{1}{4}x$.
   - The area is $A = xy = 20x - \frac{1}{4}x^2$, $0 \leq x \leq 80$, allowing for degenerate (zero) areas. Now $0 = A' = 20 - \frac{1}{2}x$ yields $x = 40$ as the lone critical value.
   - (b) Compute function values of A at the critical value $x = 40$ and the endpoints to locate where the absolute maximum occurs.

x	0	40	80
A	0	400	0

     We see that the maximum area is 400 ft^2.