1. Determine the infinite limit \( \lim_{x \to 2^-} \frac{x^2 - 2x}{x^2 - 4x + 4} \).

As \( x \to 2^- \), we have
\[
\frac{x^2 - 2x}{x^2 - 4x + 4} = \frac{(x-2)x}{(x-2)^2} = \frac{x}{x-2} \to \frac{2}{0^-} = -\infty.
\]

2. Evaluate the limit \( \lim_{u \to 2^-} \frac{\sqrt{4u+1} - 3}{u-2} \), if it exists, or state why it does not exist.

Multiply numerator and denominator by the algebraic conjugate of the numerator, then follow your nose.
\[
\lim_{u \to 2^-} \left( \frac{\sqrt{4u+1} - 3}{u-2} \cdot \frac{\sqrt{4u+1} + 3}{\sqrt{4u+1} + 3} \right) = \lim_{u \to 2^-} \frac{4u+1 - 9}{(u-2)(\sqrt{4u+1} + 3)} = \lim_{u \to 2^-} \frac{4u - 8}{(u-2)(\sqrt{4u+1} + 3)} = \lim_{u \to 2^-} \frac{4}{\sqrt{4u+1} + 3} = \frac{4}{3 + 3} = \frac{2}{6} = \frac{1}{3}.
\]

3. For what value of the constant \( c \) is the function \( f \) continuous on \((-\infty, \infty)\)?

\[
f(x) = \begin{cases} 
  cx^2 + 2x & \text{if } x < 2 \\
  x^3 - cx & \text{if } x \geq 2
\end{cases}
\]

Since \( f \) is piecewise polynomial for \( x \neq 2 \), it is continuous there. Now what about at \( x = 2 \)?
Set left and right limits equal and solve for \( c \!:
\[
\lim_{x \to 2^-} (cx^2 + 2x) = \lim_{x \to 2^+} (x^3 - cx)
\]
\[
4c + 4 = 8 - 2c
\]
\[
6c = 4
\]
\[
c = \frac{2}{3}
\]

[Thus \( f(x) = \begin{cases} 
  \frac{2}{3}x^2 + 2x & \text{if } x < 2 \\
  x^3 - \frac{2}{3}x & \text{if } x \geq 2
\end{cases} \).

Hence \( \lim_{x \to 2^-} f(x) = \frac{20}{3} = f(2) \) and \( f \) is continuous at \( x = 2 \) and therefore on \((-\infty, \infty)\).]

4. Use the Intermediate Value Theorem to show that there is a root of the equation \( x^4 + x - 3 = 0 \) in the interval \((1, 2)\).

Let \( f(x) = x^4 + x - 3 \). Since \( f \) is a polynomial, it is continuous on \( \mathbb{R} \) and thus on \([1, 2]\). Now \( f(1) = -1 \) and \( f(2) = 15 \). Since 0 is between -1 and 15, by the Intermediate Value Theorem there is a number \( c \) in \((1, 2)\) such that \( f(c) = 0 \).