1. Use the FTC1 (Fundamental Theorem of Calculus, Part 1) and the Chain Rule to compute the derivative $g'(x)$ by hand given that $g(x) = \int_1^{\cos x} (t^2 + 1)^3 \, dt$.

   - Via FTC1 and the Chain Rule, we have $g'(x) = (\cos^2 x + 1)^3 (-\sin x)$.

2. Use the substitution $u = 1 + \sqrt{x}$ to evaluate the integral $\int_1^4 \frac{1 + \sqrt{x}}{\sqrt{x}} \, dx$. Show your steps. Check your work with your calculator. (NOTE: On the rest of the integrals, just use your calculator!)

   - Let $u = 1 + \sqrt{x}$. Then $du = \frac{1}{2} x^{-1/2} \, dx$. Thus $2\, du = \sqrt{x} \, dx$.
   - When $x = 1$, we have $u = 2$; for $x = 4$, we have $u = 3$.
   - Now substitute and dispatch.

   \[
   2 \int_2^3 u^4 \, du = \frac{2}{5} u^5 \bigg|_2^3 = \frac{2}{5} (243 - 32) = \frac{422}{5} = 84.4
   \]

3. Compute the area of the region bounded by $y = x^2 - x$ and $y = 1 - x^2$.

   - Solve $x^2 - x = 1 - x^2$ to obtain $x = \pm 1$.
   - Note that the cubic curve lies above the parabola.
   - Hence the area is $\int_{-1}^{1} (1 - x^2) - (x^2 - x) \, dx$, which equals $\frac{4}{3} \approx 1.33 \text{ cm}^2$.

4. Find the volume of the solid obtained by rotating the region bounded by $x = -\pi/3$, $x = \pi/3$, $y = \cos x$, and $y = 1/2$, about the $x$-axis.

   - The volume via cross sections is $\pi \int_{-\pi/3}^{\pi/3} \cos^2 x - \left( \frac{1}{2} \right)^2 \, dx$
   - $= \frac{\pi}{12} \left( 2\pi + 3\sqrt{3} \right) \approx 3.01 \text{ cm}^3$.

5. Use the method of cylindrical shells to determine the volume of the solid obtained by rotating the region bounded by $x = y^3 - y^4$ and $x = 0$ about the line $y = 2$.

   - Solve $0 = y^3 - y^4$ to obtain $y = 0, 1$.
   - Volume via cylindrical shells is
   \[
   \int_0^1 2\pi (2 - y) (y^3 - y^4) \, dy = \frac{2\pi}{15} \approx 0.42 \text{ cm}^3.
   \]

6. A rectangular tank with its top at ground level is used to catch runoff water. The tank is 3 m wide, 4 m long and 6 m deep. How much work does it take to empty the tank by pumping the water back to ground level once the tank is full? The mass density of water is $\rho = 1000 \text{ kg/m}^3$ and the acceleration due to gravity is $g = 9.8 \text{ m/s}^2$.

   - Let the $y$-axis point vertically upward.
   - A rectangular layer of water at level $y$ (and hence at a depth of $0 - y$ from the top surface of the water) has length 4, width 3, and differential thickness $dy$. Accordingly, its volume is $dV = 12 \, dy$.
   - Herewith the march of the differentials.

   \[
   \frac{dm}{dV} = \rho \, dV = 12\rho \, dy \quad \frac{dF}{dW} = (dm) g = 12\rho g \, dy \quad \frac{dW}{dW} = (dF) (0 - y) = -12\rho gy \, dy
   \]

   - The work required to pump the water out of the tank is $W = \int dW$ or
   \[
   \int_{-6}^0 -12 (1000) \left( \frac{29}{30} \right) y \, dy = 2,116,800 \, J \approx 2.12 \times 10^6 \, J
   \]
   or approximately 2.12 megajoules.
7. Suppose that a 5000 L fuel tank takes 10 minutes to drain and that after \( t \) minutes, the volume of fuel remaining in the tank is \( V = 50(10-t)^2 \) L. What is the average volume of fuel in the tank during the time it drains?

- The average volume is \( \frac{1}{10-0} \int_{0}^{10} V \, dt \).

\[
\frac{1}{10-0} \int_{0}^{10} 50(10-t)^2 \, dt = \frac{5000}{3} \approx 1666.67 \text{ L}.
\]

8. Find the arc length of the curve \( y = \frac{1}{3} (x^2 + 2)^{3/2} \) from \( x = 0 \) to \( x = 3 \).

- Now \( \frac{dy}{dx} = \frac{1}{2} \left( x^2 + 2 \right)^{1/2} \cdot 2x = x\sqrt{x^2 + 2} \).

- The arc length is \( L = \int_{a}^{b} \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx \) or

\[
= \int_{0}^{3} \frac{x^2 + 1}{\sqrt{1 + \left( x\sqrt{x^2 + 2} \right)^2}} \, dx = 12 \text{ cm}.
\]

9. Find the surface area generated by rotating the curve \( x = \sqrt{y} \), \( 0 \leq y \leq 2 \), about the \( y \)-axis.

- Now \( \frac{dx}{dy} = \frac{1}{2} y^{-1/2} = \frac{1}{2\sqrt{y}} \).

- The surface area is \( \int 2\pi r \, ds = \int_{a}^{b} 2\pi x \sqrt{1 + \left( \frac{dx}{dy} \right)^2} \, dy \).

\[
= \int_{0}^{2} 2\pi \cdot \sqrt{y} \sqrt{1 + \left( \frac{1}{2\sqrt{y}} \right)^2} \, dy
= \frac{13\pi}{3} \approx 13.61 \text{ cm}^2.
\]