Section 11.1

1. Consider the origin \( O(0,0,0) \) and point \( P(1,5,6) \) in 3D-space. Draw a rectangular box with these points as opposite vertices of a diagonal through the box and faces parallel to the coordinate planes. Label the boxes vertices by projection onto coordinate planes.

2. Find the lengths of sides of the triangle with vertices \( P(5,-1,-1), Q(7,0,1), R(8,-2,-1) \). Is it a right triangle? Is it an isosceles triangle?

3. Find an equation of the sphere with center \( C(3,6,4) \) and radius \( r = 10 \). Describe the intersection of this sphere with the \( xz \)-plane.

4. Find an equation of the sphere with one of whose diameters has endpoints \( A(5,4,5) \) and \( B(7,8,7) \).

5. Find equations of the three spheres with center \( C(2,-5,4) \) that touch each coordinate plane in a single point.

6. By completing squares, find an equation of the sphere \( x^2 + y^2 + z^2 + 4x - 2y - 2z = 19 \), then give the center and radius of the sphere.

7. Do the same for \( x^2 + y^2 + z^2 + 10x - 8y + 2z + 38 = 0 \).

8. Describe what these equation(s) represent in \( \mathbb{R}^3 \), \( xyz \)-space. [Also describe what the first one represents in \( \mathbb{R}^2 \), the \( xy \)-plane.]

   (a) \( x = 6 \)
   
   (b) \( y + 3x = 4 \)
   
   (c) \( z - 4y = 8 \)
   
   (d) \( \) the set of points such that \( y = 4 \) and \( z = 8 \)

Section 11.2

1. Given points \( A(0,2,4) \) and \( B(3,2,-2) \), compute \( \mathbf{v} = \overrightarrow{AB} \), the vector from \( A \) to \( B \). In \( xyz \)-space, draw \( \mathbf{v} \) and its equivalent position vector.

2. Find the vector sum of \( \mathbf{a} = [3,0,3] \) and \( \mathbf{b} = [0,6,0] \). Illustrate geometrically.

3. Find a unit vector \( \mathbf{v} \) in the same direction as \( \mathbf{v} = [8,-1,4] \).

4. Let \( \mathbf{a} = [4,-6,4] \) and \( \mathbf{b} = [0,2,-1] \). Find \( \mathbf{a} + \mathbf{b} \), \( 2\mathbf{a} + 3\mathbf{b} \), \( ||\mathbf{a}|| \), and \( ||\mathbf{a} - \mathbf{b}|| \).

5. Find a vector \( \mathbf{w} \) that has the same direction as \( \mathbf{v} = [-4,6,2] \), but has magnitude (length) 6.

6. Are the following expressions meaningful or meaningless? In each case, explain why.

   (a) \( (\mathbf{a} \cdot \mathbf{b}) \cdot \mathbf{c} \)
   
   (b) \( (\mathbf{a} \cdot \mathbf{b}) \mathbf{c} \)
   
   (c) \( ||\mathbf{a}|| (\mathbf{b} \cdot \mathbf{c}) \)
   
   (d) \( \mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) \)
   
   (e) \( \mathbf{a} \cdot \mathbf{b} + \mathbf{c} \)
   
   (f) \( ||\mathbf{a}|| (\mathbf{b} + \mathbf{c}) \)

7. Find the angle \( \theta \) between vectors \( \mathbf{a} = [5,-1,7] \) and \( \mathbf{b} = [-2,6,3] \), both exactly and approximately (to the nearest degree).

8. A triangle has vertices \( A(0,1,1), B(-1,2,4), \) and \( C(2,5,-1) \). Find its angles to the nearest degree.

9. Let \( \mathbf{a} = [2,3,-6] \) and \( \mathbf{b} = [-1,-1,4] \). Find \( \text{comp}_\mathbf{a} \mathbf{b} \) and \( \text{proj}_\mathbf{a} \mathbf{b} \), scalar and vector projections of \( \mathbf{b} \) onto \( \mathbf{a} \).

10. In each case, determine whether the pair of vectors is orthogonal, parallel, or neither.

    (a) \( \mathbf{a} = [-7,4,7], \mathbf{b} = [8,-7,1] \)
    
    (b) \( \mathbf{a} = [2,6], \mathbf{b} = [-3,1] \)
    
    (c) \( \mathbf{a} = [-1,4,3], \mathbf{b} = [9,3,-1] \)
    
    (d) \( \mathbf{a} = [2,6,-6], \mathbf{b} = [-3,-9,9] \)

Section 11.3

1. Are the following expressions meaningful or meaningless? In each case, explain why.

   (a) \( \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) \)
   
   (b) \( \mathbf{a} \times (\mathbf{b} \cdot \mathbf{c}) \)
   
   (c) \( \mathbf{a} \times (\mathbf{b} \times \mathbf{c}) \)
   
   (d) \( \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) \)
   
   (e) \( (\mathbf{a} \cdot \mathbf{b}) \times (\mathbf{c} \cdot \mathbf{d}) \)
   
   (f) \( (\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d}) \)

2. Find \( \mathbf{c} = \mathbf{a} \times \mathbf{b} \) given \( \mathbf{a} = [1,1,-1] \) and \( \mathbf{b} = [3,7,9] \). Verify that \( \mathbf{c} \) is orthogonal to \( \mathbf{a} \) and \( \mathbf{b} \).
3. Given \( \mathbf{a} = [0, 1, 9] \) and \( \mathbf{b} = [3, -1, 3] \), find \( \mathbf{a} \times \mathbf{b} \) and \( \mathbf{b} \times \mathbf{a} \).

4. Position vectors \( \mathbf{u} \) and \( \mathbf{v} \) in the plane have magnitudes 2 and 5, respectively. They point north and N60°E, respectively. Determine whether \( \mathbf{u} \times \mathbf{v} \) is directed into the plane or out of it. Then find its magnitude.

5. Do Stewart’s 672/4 in the lecture handout, but with the magnitudes of \( \mathbf{a} \) and \( \mathbf{b} \) each equal to 6.

6. Find a nonzero vector orthogonal to the plane determined by points \( P(3, 0, 3), Q(-2, 1, 4), \) and \( R(5, 2, 5) \). Then find the area of triangle \( PQR \).

7. Find the area of the parallelogram in space with vertices \( K(3, 3, 1), L(3, 5, 2), M(7, 9, 2), \) and \( N(7, 7, 1) \).

8. Find the volume of the parallelepiped (sheared box) determined by the vectors \( \mathbf{a} = [1, 2, 2], \mathbf{b} = [-1, 1, 2], \) and \( \mathbf{c} = [5, 1, 5] \).

9. Find the volume of the parallelepiped with adjacent edges \( \overrightarrow{PQ}, \overrightarrow{PR}, \overrightarrow{PS} \) connecting the points \( P(1, 0, 3), Q(-4, 1, 7), R(3, 3, 2), S(-1, 4, 4) \).

10. Use a scalar triple product or \( 3 \times 3 \) determinant to see if these vectors are coplanar (lie in the same plane).

\[
\mathbf{u} = [1, 5, -3] \quad \mathbf{v} = [4, -1, 0] \quad \mathbf{w} = [8, 14, -9]
\]

**Section 11.4**

1. Find vector and parametric equations for the line through the point \( P(8, -5, 4) \) that is parallel to the vector \( \mathbf{v} = [1, 5, -\frac{2}{3}] \). Use the parameter \( t \).

2. Find parametric equations for the line passing through the points \( A(10, 3, 1) \) and \( B(6, 6, -1) \).

3. Find symmetric equations for the line that passes through the point \( A(1, 2, -7) \) and is parallel to the vector \( \mathbf{v} = [-1, 2, -3] \). Find the points at which the line intersects the coordinate planes.

4. Find vector and parametric equations for the line through \( A(0, 12, -7) \) that is parallel to the line given by \( [x, y, z] = [-1 + 3t, 6 - 3t, 3 + 7t] \).

5. Find parametric equations for the line through \( P(1, 2, 8) \) that is perpendicular to the plane \( x - y + 4z = 2 \).

6. Find the point at which the lines intersect. Then find an equation for the plane determined by the lines.

\[
f(t) = [1, 2, 0] + t[2, -2, 2] \\
g(s) = [3, 0, 2] + s[-2, 2, 0]
\]

7. Find an equation of the plane through the point \( P(7, -7, -9) \) that is parallel to the plane with equation \( 5x - y - z = 7 \).

8. Find an equation of the plane through the points \( P(2, -1, 3), Q(7, 3, 6), \) and \( R(-2, -2, -3) \).

9. Find an equation of the plane through \( P(3, 4, 5) \) that contains the line \( x = 5t, y = 3 + t, z = 4 - t \).

10. Find the point at which the line \( x = 4 - t, y = 3 + t, z = 2t \) intersects the plane \( x - y + 4z = 7 \).

11. Find parametric equations for the line of intersection of the planes \( 4x - 2y + z = 2 \) and \( 2x + y - 4z = 3 \). Then find the angle between the planes in degrees.

12. Find the (perpendicular) distance from the point \( P(1, -9, 4) \) to the plane \( 3x + 2y + 6z = 5 \).

13. Find the distance between these parallel planes.

\[
3x - 4y + z = 15 \\
6x - 8y + 2z = 3
\]