By default, your calculator is set to AUTO mode. If it can compute an answer exactly it will. Otherwise, it will return a decimal approximation automatically. Even when an exact answer is returned, please additionally compute an approximation as a courtesy to your instructor. Lengths are in cm, areas in cm$^2$, and volumes in cm$^3$.

**Section 13.1**

Don’t bother with specific Riemann sums. Just dispatch integrals directly. If your calculator needs to approximate, it will return a result automatically that is far more accurate that anything you can do by hand.

1. Compute the volume of the solid lying below $z = xy$ and above rectangle $\{(x,y): 4 \leq x \leq 10, 0 \leq y \leq 4\}$.

2. Same drill for the solid lying below the surface $z = 9x + 4y^2$ and above rectangle $R = [0, 2] \times [0, 4]$, which is Cartesian set product shorthand for the set $\{(x,y): 0 \leq x \leq 2, 0 \leq y \leq 4\}$.

3. If $R = [-2, 2] \times [-1, 1]$, compute $\iint_R y^2 - 2x^2 \, dA$.

4. Evaluate $\iint_R 4 \, dA$ directly as well as identifying it as the volume of a solid. Here $R$ is the rectangular region $\{(x,y): -2 \leq x \leq 2, 3 \leq y \leq 8\}$.

5. Same as #4 for $\iint_R 8 - x \, dA$ with $R = \{(x,y): 0 \leq x \leq 8, 0 \leq y \leq 5\}$.

**Section 13.2**

To do multiple integrals by hand, work from the inside out, repeatedly using the Fundamental Theorem of Calculus, Part 2. That said, computation of integrals is automatic on your calculator, which is what I would recommend for speed, accuracy, and less mental wear and tear.

1. Find $\int_0^3 \int_0^2 f(x,y) \, dx \, dy$ and $\int_0^3 \int_0^2 f(x,y) \, dy$ via partial integration (treating the other variable as a constant). Here $f(x,y) = y + xe^y$.

2. Calculate the double integral $\iint_R 18x^2y^3 - 15y^4 \, dA$, where $R = \{(x,y): 0 \leq x \leq 3, 0 \leq y \leq 1\}$.

3. Calculate the iterated integral $\int_0^9 \int_0^{\pi/2} y + y^2 \cos x \, dx \, dy$.

4. Compute the iterated integral $\int_0^{10} \int_0^{\pi/2} 3x \sin y \, dy \, dx$.

5. Calculate the double integral $\iint_R 3\cos(x + 2y) \, dA$, where $R = [0, 5\pi] \times [0, \frac{1}{2}\pi]$.

6. Compute the iterated integral $\int_0^1 \int_0^1 3\sqrt{s+t} \, ds \, dt$.

7. Same as #6 for $\int_0^2 \int_0^6 5e^{t+3y} \, dx \, dy$.

8. Compute $\iint_R 7x y e^{x^2 y} \, dA$ for $R = [0, 1] \times [0, 3]$.

9. Calculate $\iint_R \frac{3xy^2}{x^2 + 1} \, dA$, where $R = \{(x,y): 0 \leq x \leq 1, -2 \leq y \leq 2\}$.

10. Find the volume of the solid that lies under the plane $4x + 10y - 2z + 15 = 0$ and above the rectangle $R = (x,y): -1 \leq x \leq 2, -1 \leq y \leq 1$.

11. Same for the solid under the hyperbolic paraboloid $z = 18 + x^2 - y^2$ and above rectangle $[-4, 4] \times [0, 4]$.

12. Find the volume of the solid in the first octant bounded by the parabolic cylinder $z = 16 - x^2$ and the plane $y = 4$.

**Section 13.3**

1. Evaluate the iterated integral $\int_0^3 \int_0^2 5xy \, dx \, dy$.

2. Same for $\int_0^1 \int_0^s 9 + 18y \, dy \, dx$.

3. Same for $\int_0^1 \int_0^2 \cos(s^8) \, dt \, ds$.

4. Evaluate the double integral $\iint_D 3x \cos y \, dA$ where $D$ is bounded by $y = 0$, $y = x^2$, and $x = 3$.

5. Evaluate $\iint_D 7y \, dA$, where $D$ is the region $D = \{(x,y): -y - 2 \leq x \leq y, -1 \leq y \leq 1\}$.

6. Evaluate the integral $\iint_D \frac{5y}{6x^2 + 1} \, dA$, where $D = \{(x,y): 0 \leq y \leq x^2, 0 \leq x \leq 1\}$.

7. Find the volume of the solid under the plane $3x + 3y - z = 0$ and above the region bounded by $y = x$ and $y = x^4$.

8. Same for the solid under the surface $z = 7xy$ & above the triangle with vertices $A(1,1)$, $B(4,1)$, and $C(1,2)$.

9. Find the volume enclosed by the paraboloid $z = 7x^2 + 4y^2$ and the planes $x = 0$, $y = 4$, $y = x$, and $z = 0$.

10. For the integral $\int_0^2 \int_0^4 f(x,y) \, dA$, sketch the region of integration in the $xy$-plane. Then set up an equivalent integral with the order of integration reversed.
11. Same as #10 for \( \int_0^1 \int_0^{\ln x} f(x, y) \, dy \, dx \).

12. Evaluate the integral \( \int_0^4 \int_{\frac{-\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} 5e^{x^2} \, dx \, dy \) exactly by reversing the order of integration.

13. Same as #12 for \( \int_0^6 \int_{\sqrt{3}}^{2} \frac{2}{x^2 + 1} \, dy \, dx \).

14. Same for \( \int_0^4 \int_{\frac{\pi}{4}}^{\frac{4\pi}{3}} 2e^{x^2} \, dx \, dy \).

**Section 13.4**

When graphing polar curves, use your calculator!

1. Find Cartesian coordinates given polar coordinates, then plot the points.
   (a) \((4, \pi)\)  
   (b) \((-2, \frac{3\pi}{2})\)  
   (c) \((-4, \frac{3\pi}{2})\)

2. Find polar coordinates given Cartesian coordinates.
   (a) For \((4, -4)\), specify \((r, \theta)\) with \(r > 0\) and \(0 \leq \theta < 2\pi\), then with \(r < 0\) and \(0 \leq \theta < 2\pi\).
   (b) Same for \((-1, \sqrt{3})\).

3. Sketch the region in the plane for which \(2 < r < 5\) and \(\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}\).

4. Sketch the polar curve \(r = \theta\), \(0 \leq \theta \leq 2\pi\).

5. Sketch the curve with polar equation \(\theta = -\pi/6\).

6. Graph the curve with polar equation \(r = \sin \theta\).

7. Graph \(r = -7\cos \theta\).

8. Graph \(r = 2\sin 4\theta\).

9. Graph \(r = 6\cos 2\theta\).

10. Graph \(r = 4 + 3\sin \theta\).

11. Graph \(r = 1 - 3\cos \theta\).

12. Find a polar equation for the curve represented by Cartesian equation \(x^2 + y^2 = 9\).

13. Same as #12 for \(xy = 4\).

14. Find a Cartesian equation for the curve \(r = 2\sin \theta\). Identify the curve.

15. Same as #14 for \(r = \csc \theta\).

**Section 13.5**

The area differential is \(dA = dx \, dy = dy \, dx = r \, dr \, d\theta\).

1. Evaluate the integral \(\int_D e^{-x^2 - y^2} \, dA\) by switching to polar coordinates. Here \(D\) is the region that lies to the left of the \(y\)-axis and between circles \(x^2 + y^2 = 1\) and \(x^2 + y^2 = 49\).

2. Compute the integral \(\int_D \sqrt{16 - x^2 - y^2} \, dA\) by converting to polar coordinates.

3. Compute \(\int_D \sqrt{16 - x^2 - y^2} \, dA\) by switching to polar coordinates. Here \(D = \{(x, y) : x^2 + y^2 \leq 16, x \geq 0\}\).

4. Evaluate \(\int_0^\pi \int_0^{\frac{\sqrt{64 - r^2}}{8}} 8\sqrt{x^2 + y^2} \, dy \, dx\) by switching to polar coordinates.

5. Evaluate the integral \(\int_0^1 \int_{\frac{\sqrt{6 - r^2}}{2}}^{\frac{\sqrt{2 - r^2}}{2}} 5(x + y) \, dy \, dx\) by converting to polar coordinates.

6. Use polar coordinates to find the volume of the solid under the cone \(z = \sqrt{x^2 + y^2}\) and above the circular disk \(x^2 + y^2 \leq 16\).

7. Same as #6 for the solid above the \(xy\)-plane that lies below the ellipsoid \(4x^2 + 4y^2 + z^2 = 64\) and inside the circular cylinder \(x^2 + y^2 = 1\).

8. Same for the solid below the paraboloid \(z = 18 - 2x^2 - 2y^2\) and above the \(xy\)-plane.

9. Same for the solid bounded by paraboloids \(z = 4x^2 + 4y^2\) and \(z = 4 - x^2 - y^2\).

10. Use a double integral in polar coordinates to find the area of one loop of the rose \(r = 7\cos 3\theta\).

11. Same for the region inside the offset circle \(r = 8\sin \theta\) and outside the circle \(r = 4\) centered at the origin.

**Section 13.6**

1. Electric charge is distributed over the rectangular region \(2 \leq x \leq 5, 0 \leq y \leq 2\), with charge density \(\sigma(x, y) = 2xy + x^2\) measured in coulombs per m\(^2\). Find the total charge on the region.

2. Find the mass and center of mass of the lamina (flat plate) \(D = \{(x, y) : 0 \leq x \leq 2, -1 \leq y \leq 1\}\) that has variable density \(\delta(x, y) = 2xy^2\).

3. Same as #2 for the triangular region \(D\) with vertices \((0, 0), (2, 1), (0, 3)\) and density \(\delta(x, y) = 2(x + y)\).
4. Same drill for the region $D$ bounded by $y = \sqrt{x}$, $y = 0$, $x = 1$ with density $\delta(x, y) = 16x$.

5. Find the center of mass of the portion of the disk $x^2 + y^2 \leq 4$ in the first quadrant whose density $\delta$ at any point is proportional to the square of the distance from the origin.

Section 13.7

1. Find the surface area of the part of the plane $z = 2 + 5x + 2y$ that lies above the rectangle $[0, 9] \times [1, 6]$.

2. Same as #1 for part of the plane $2x + 4y + z = 8$ that lies inside the cylinder $x^2 + y^2 = 25$.

3. Same for the part of the cylinder $y^2 + z^2 = 9$ that lies above the rectangle with vertices $(0, 0), (5, 0), (5, 2)$, and $(0, 2)$.

4. Same for part of the hyperbolic paraboloid $z = y^2 - x^2$ between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 25$.

5. Find the area of the part of the surface $z = xy$ that lies within the cylinder $x^2 + y^2 = 4$.

Section 13.8

To do multiple integrals by hand, work from the inside out, repeatedly using the Fundamental Theorem of Calculus, Part 2. That said, computation of integrals is automatic on your calculator, which is what I would recommend for speed, accuracy, and less mental wear and tear.

1. Evaluate the triple integral $\iiint_E xz - y^3 \,dz \,dy \,dx$ using three different orders of integration (there are six) just to show that “All roads lead to Rome!” Here $E$ is the solid $\{(x, y, z) : -1 \leq x \leq 1, 0 \leq y \leq 4, 0 \leq z \leq 3\}$.

2. Compute $\int_0^2 \int_x^2 \int_0^8 \,8xyz \,dz \,dy \,dx$.

3. Evaluate $\iiint_E y \,dV$, where $E$ is bounded by the planes $x = 0, y = 0$, $z = 0$, and $x + y + z = 1$ in the first octant.

4. Compute the triple integral $\iiint_E 5x \,dV$, where $E$ is bounded by the circular paraboloid $x = 5y^2 + 5z^2$ and the plane $x = 5$.

5. Use a triple integral to find the volume of the tetrahedron in the first octant bounded by the coordinate planes and the plane $3x + y + z = 5$.

6. Write the integral $\int_0^1 \int_{x^2}^1 \int_{y^3}^1 f(x, y, z) \,dz \,dy \,dx$ in 5 other orders, permuting the differentials.

7. Find the mass and center of mass of the solid $E$ that is bounded by the parabolic cylinder $z = 1 - y^2$ and the planes $x + 3z = 3, x = 0$, and $z = 0$. The density of the solid is constant, $\delta = 3$.

Section 13.9

1. Find rectangular coordinates of the point whose cylindrical coordinates are $(r, \theta, z) = (3, 0, 1)$. Same for $(r, \theta, z) = (5, -\frac{5}{6}\pi, 3)$.

2. Find cylindrical coordinates of the point whose rectangular coordinates are $(x, y, z) = (5, -5, 2)$. Same for $(x, y, z) = (-4, -4\sqrt{3}, 4)$.

3. Write the Cartesian equation $7z = 3x^2 + 3y^2$ using cylindrical coordinates. Same for $5x^2 + 5y^2 = 8$.

4. What solid is described by these inequalities?

$$0 \leq r \leq 5, \quad -\pi \leq \theta \leq \pi, \quad -\sqrt{25 - r^2} \leq z \leq \sqrt{25 - r^2}$$

5. Find rectangular coordinates of the point whose spherical coordinates are $(\rho, \theta, \phi) = (1, \frac{3}{2}\pi, \frac{1}{2}\pi)$. Same for $(\rho, \theta, \phi) = (2, \frac{1}{2}\pi, \frac{1}{2}\pi)$.

6. Find spherical coordinates of the point whose rectangular coordinates are $(x, y, z) = (0, 4\sqrt{3}, 4)$. Same for $(x, y, z) = (-5, 5, 5\sqrt{6})$.

7. Write the Cartesian equation $5x^2 - y + 5y^2 + 5z^2 = 0$ using spherical coordinates. Same for $x + 6y + 4z = 1$.

8. A solid lies above the cone $4z = \sqrt{x^2 + y^2}$ and outside the offset sphere $x^2 + y^2 + z^2 = 62$. Give a description of the solid in terms of spherical inequalities.

Section 13.10

The volume differential is

$$dV = dx \,dy \,dz = r \,dz \,dr \,d\theta = \rho^2 \sin \phi \,d\rho \,d\phi \,d\theta.$$
4. Evaluate \( \int_{-2}^{2} \int_{0}^{\sqrt{4-x^2}} \int_{0}^{\sqrt{4-x^2-y^2}} \sqrt{x^2+y^2} \, dz \, dy \, dx \)
by changing to cylindrical coordinates.

5. Compute the integral \( \iiint_B (x^2 + y^2 + z^2)^2 \, dV \), where
\( B \) is the solid spherical ball with center the origin and
radius 1. Use spherical coordinates.

6. Evaluate \( \int_{0}^{4} \int_{0}^{\sqrt{16-x^2}} \int_{0}^{\sqrt{32-x^2-y^2}} xy \, dz \, dy \, dx \)
by changing to spherical coordinates.

7. Using cylindrical and/or spherical coordinates, find
the volume and centroid (center of mass with constant
density \( \delta = k \)) of the solid \( E \) that lies above the cone
\( z = \sqrt{x^2+y^2} \) and below the sphere \( x^2 + y^2 + z^2 = 49 \).

**Section 13.11**

1. Find the Jacobian matrix and determinant of the
transformation \( x = 5u + v, \ y = 4u + 5v \).

2. Find the image of \( S = \{(u,v) : 0 \leq u \leq 6, 0 \leq v \leq 5\} \)
for the transformation \( x = 2u + 3v, \ y = u - v \). Sketch.

3. Use the transformation \( x = 2u, \ y = 3v \), to evaluate
\( \iint_D 4x^2 \, dA \), where \( D \) is the region bounded by the
ellipse \( 9x^2 + 4y^2 = 36 \).

4. Use the transformation \( x = u/v, \ y = v \), to evaluate
\( \iint_D 3xy \, dA \), where \( D \) is the region in the first quadrant
bounded by the lines \( y = \frac{4}{3}x \) and \( y = 4x \) along with the
hyperbolas \( xy = \frac{4}{3} \) and \( xy = 4 \).

5. Use an appropriate change of variables to evaluate the
integral \( \iint_D 9 \cos \left( 9 \left( \frac{y-x}{y+x} \right) \right) \, dA \), where \( D \) is the
trapezoidal region whose vertices are \((7,0),\ (9,0),\ (0,9),\ \)and \((0,7)\).

6. Same drill for \( \iint_D 8 \sin (98x^2 + 50y^2) \, dA \), where \( D \) is the
region in the first quadrant bounded by the ellipse
\( 49x^2 + 25y^2 = 1 \) and the coordinate axes.

**NOTES**