Section 14.1

1. Match vector fields $\mathbf{F}$ with plots on your WA screens, 
   (a) $[y, x]$ (b) $[1, \sin y]$ (c) $[x - 2, x + 1]$ (d) $[y, 1/x]$
2. Find & sketch gradient vector field for $f = x^2 - 4y$.
3. Same as #2 for $f = xe^{3xy}$.
4. Find the gradient vector field of $f = \tan(3x - 5y)$.
5. Find the gradient vector field of $f = \cos(4y/z)$.

Section 14.2

1. Evaluate line integral $\int_C xy^2 \, ds$, where $C$ is right half of circle $x^2 + y^2 = 4$ oriented counterclockwise.
2. Same as #1 for $\int_C xy \, dx + (x - y) \, dy$, where $C$ consists of line segments from $(0, 0)$ to $(5, 0)$ to $(6, 2)$.
3. Same as #1 for $\int_C xyz \, ds$, where $C$ is parameterized as $g = [4 \sin t, t, -4 \cos t]$, $0 \leq t \leq \pi$.
4. Evaluate $\int_C \mathbf{w} \cdot \mathbf{d}g$, where $\mathbf{w} = [\sin x, \cos y, xz]$ and $\mathbf{g} = [t^3, -t^3, t]$, $0 \leq t \leq 1$.
5. A thin wire is bent into the shape of a semicircle $x^2 + y^2 = 36$, $x \geq 0$. If the linear density is a constant, $\delta = k$, find the mass and center of mass of the wire.
6. Find the work done by the force field $\mathbf{w} = [x \sin y, y]$ on a particle that moves along the parabola $y = x^2$, from $(-2, 4)$ to $(-1, 1)$.

Section 14.3

1. Is $\mathbf{w} = [2x - 6y, -6x + 14y - 5]$ is a conservative vector field? If so, find $f$ such that $\mathbf{w} = \nabla f$. (The function $f$ is a potential function for $\mathbf{w}$.)
2. Same as #1 for $\mathbf{w} = [ye^x + \sin y, e^x + x \cos y]$. 
3. Same as #1 for $\mathbf{w} = [3x^2 - 3y^2, 6xy + 3]$.
4. Let $C$ be the arc of the parabola $y = 4x^2$ from $(-1, 4)$ to $(1, 4)$ and $\mathbf{w} = [x^2, y^2]$. Find a potential function $f$ for $\mathbf{w}$. Then use the FTLI (Fundamental Theorem for Line Integrals) to compute $\int_C \mathbf{w} \cdot \mathbf{d}g$, where $\mathbf{g}$ is any parameterization of $C$, which is immaterial since you are not going to use it!

Section 14.4

In this section closed curves are positively oriented; i.e., traversed clockwise.

1. Use Green’s Theorem to evaluate the line integral $\int_C xy^2 \, dx + 3x^2y \, dy$ along the triangle $C$ with vertices $A(0, 0)$, $B(3, 3)$, $C(3, 6)$.
2. Use Green’s Theorem to evaluate the line integral $\int_C (3y + 5e^x) \, dx + (10x + 7 \cos y^2) \, dy$ where $C$ is the boundary of the region enclosed by the parabolas $y = x^2$ and $x = y^2$.
3. Same as #2 for $\int_C 9y^3 \, dx - 9x^3 \, dy$ where $C$ is the circle $x^2 + y^2 = 4$.
4. Use Green’s Theorem to find the work done by the force field $\mathbf{w} = [x(x + y), xy^2]$ moving a particle from the origin along the $x$-axis to $(3, 0)$, then along the line segment to $(0, 3)$, and back to the origin along the $y$-axis.
5. A particle starts at the point $(-2, 0)$, moves along the $x$-axis to $(2, 0)$, and then along the semicircle $y = \sqrt{4 - x^2}$ to the starting point. Use Green’s Theorem to find the work done on this particle by the force field $\mathbf{w} = [x, x^3 + 3xy^2]$.

Section 14.5

1. Find the curl and the divergence of the vector field $\mathbf{w} = [x^2yz, xy^2z, xyz^2]$.
2. Same as #1 for $\mathbf{w} = [xye^x, 0, yze^x]$.
3. Is $\mathbf{w} = [6xy, 3x^2 + 4yz, 2y^2]$ is a conservative vector field? If so, find $f$ such that $\mathbf{w} = \nabla f$.
4. Same as #3 for $\mathbf{w} = [ye^{-x}, e^{-x}, 3z]$. 

5. Let \( \mathbf{F} \) and \( \mathbf{G} \) be vector fields. Assuming that appropriate partial derivatives exist and are continuous, find an identical expression for \( \text{curl} (\mathbf{F} + \mathbf{G}) \) from the choices below.

(a) \( \text{div} \mathbf{F} + \text{div} \mathbf{G} \)
(b) \( \text{curl} (\mathbf{F}) + \text{curl} (\mathbf{G}) \)
(c) \( \mathbf{G} \cdot \text{curl} (\mathbf{F}) - \mathbf{F} \cdot \text{curl} (\mathbf{G}) \)
(d) none of the preceding

6. If \( f \) is a scalar field and \( \mathbf{G} \) is a vector field, then \( f \mathbf{G} \) is defined as \( (f \mathbf{G})(x, y, z) = f(x, y, z) \mathbf{G}(x, y, z) \), i.e., via scalar multiplication. Find an identical expression for \( \text{div} (f \mathbf{G}) \), assuming appropriate partial derivatives exist and are continuous.

(a) \( \text{grad} (\text{div} \mathbf{G}) - \nabla^2 \mathbf{G} \)
(b) \( f \text{curl} \mathbf{G} + (\nabla f) \times \mathbf{G} \)
(c) \( f \text{div} \mathbf{G} + \mathbf{G} \cdot \nabla f \)
(d) none of the preceding

7. Let \( f \) be a scalar field and \( \mathbf{G} \) a vector field. Describe each of the following expressions as a scalar field, a vector field, or not meaningful.

(a) \( \text{curl} f \)
(b) \( \text{grad} f \)
(c) \( \text{div} \mathbf{G} \)
(d) \( \text{curl} (\text{grad} f) \)
(e) \( \text{grad} \mathbf{G} \)
(f) \( \text{grad} (\text{div} \mathbf{G}) \)
(g) \( \text{div} (\text{grad} f) \)
(h) \( \text{grad} (\text{div} f) \)
(i) \( \text{curl} (\text{curl} \mathbf{G}) \)
(j) \( \text{div} (\text{div} \mathbf{G}) \)
(k) \( (\text{grad} f) \times (\text{div} \mathbf{G}) \)
(l) \( \text{div} (\text{curl} (\text{grad} f)) \)

### Section 14.6

In parametric representations of surfaces below, the three slots in the vector represent spatial coordinates \( x, y, \) and \( z \), respectively. I give representations that can be plotted via the 3D parametric plotting capability on your calculator. (MATLAB of course is better.)

1. Find a parametric representation for the part of the cylinder \( y^2 + z^2 = 36 \) that lies between the planes \( x = 0 \) and \( x = 5 \).

### Section 14.7

1. Evaluate the surface integral \( \iint_S yz \, dS \) where \( S \) is the part of the plane \( x + y + z = 7 \) in the first octant.

2. Same as #1 for \( \iint_S z + x^2 y \, dS \) where \( S \) is the part of the cylinder \( y^2 + z^2 = 16 \) that lies between the planes \( x = 0 \) and \( x = 9 \) in the first octant.

3. Same as #1 for \( \iint_S zdS \) where \( S \) is the surface \( x = y + 3z^2 \), \( 0 \leq y \leq 1 \), \( 0 \leq z \leq 2 \).

4. Same as #1 for \( \iint_S \sqrt{1 + x^2 + y^2} \, dS \) where \( S \) is the helicoid \( [ucost, usint, t] \), \( 0 \leq t \leq 5\pi \), \( 0 \leq u \leq 5 \).

5. Same as #1 for \( \iint_S x^2z + y^2z \, dS \) where \( S \) is the hemisphere \( x^2 + y^2 + z^2 = 4 \), \( z \geq 0 \).

6. Evaluate \( \iint_S \mathbf{w} \cdot d\mathbf{S} \) for \( \mathbf{w} = [xy, yz, zx] \) where \( S \) is the part of the paraboloid \( z = 8 - x^2 - y^2 \) with upward orientation lying above square \( 0 \leq x \leq 1 \), \( 0 \leq y \leq 1 \).

7. Same as #6 for \( \mathbf{w} = [x, y, 4] \) where \( S \) is the boundary (with outward orientation) of the solid enclosed by the cylinder \( x^2 + z^2 = 1 \) and the planes \( y = 0 \) and \( x + y = 3 \).

8. Find the mass of a funnel in the shape of a cone \( z = \sqrt{x^2 + y^2} \), \( 1 \leq z \leq 3 \) if its density function is \( \delta = 6 - z \).
9. The temperature at the point \((x, y, z)\) in a substance with conductivity \(K = 5.5\) is \(u = 5y^2 + 5z^2\). Find the rate of heat flow inward across the cylindrical surface \(y^2 + z^2 = 7, 0 \leq x \leq 5\).

**Section 14.8**

1. Use Stokes’ Theorem to evaluate \(\int_S \text{curl } \mathbf{w} \cdot d\mathbf{S}\) where \(\mathbf{w} = [2ycosz, e^x sinz, xe^y]\) and \(S\) is the hemisphere \(x^2 + y^2 + z^2 = 9, z \geq 0\), oriented upward.

2. Same as #1 for \(\mathbf{w} = [x^2z^2, y^2z^2, xyz]\) where \(S\) is the part of the paraboloid \(z = x^2 + y^2\) that lies inside the cylinder \(x^2 + y^2 = 25\), oriented upward.

3. Same as #1 for \(\mathbf{w} = [xyz, xy, x^2yz]\) where \(S\) consists of the top and four sides (but not the bottom) of the cube with vertices \((\pm 8, \pm 8, \pm 8)\), oriented outward.

4. Use Stokes’ Theorem to evaluate \(\int_C \mathbf{w} \cdot d\mathbf{g}\) with \(\mathbf{w} = [e^{-x}, e^x, e^y]\) and \(C\) is the boundary \(g\) of the part of the plane \(2x + y + 2z = 2\) in the first octant oriented counterclockwise as viewed from above.

5. Same as #4 with \(\mathbf{w} = [x + y^2, y + z^2, z + x^2]\) and \(C\) is the triangle with vertices \((5, 0, 0)\), \((0, 5, 0)\), and \((0, 0, 5)\), oriented counterclockwise as viewed from above.

6. Same as #4 with \(\mathbf{w} = [yz, 8xz, e^y]\) and \(C\) is the circle \(x^2 + y^2 = 4, z = 7\), oriented counterclockwise as viewed from above.

7. Same as #4 with \(\mathbf{w} = [xy, 5z, 7y]\) and \(C\) is the curve of intersection of the plane \(x + z = 2\) and the cylinder \(x^2 + y^2 = 9\), oriented counterclockwise as viewed from above.

8. A particle moves along line segments from the origin to the points \((3, 0, 0), (3, 2, 1), (0, 2, 1)\), and back to the origin under the influence of the force field \(\mathbf{w} = [z^2, 3xy, 4y^2]\). Find the work done.

**Section 14.9**

1. Verify that the Divergence Theorem is true for the vector field \(\mathbf{w} = [2x, xy, 3xz]\) on the region \(E\), the solid cube bounded by the planes \(x = 0, x = 2, y = 0, y = 2, z = 0,\) and \(z = 2\). Give the flux.

2. Use the Divergence Theorem to calculate the surface integral \(\iint_S \mathbf{w} \cdot d\mathbf{S}\); that is, calculate the flux of \(\mathbf{w}\) across \(S\). Here \(\mathbf{w} = [x^2z^3, 2xyz^3, xz^4]\) and \(S\) is the surface of the solid box with vertices \((\pm 3, \pm 1, \pm 3)\).