• In Calculus 2, the average value of \( f(x) \) on the interval \( I = [a, b] \) is

\[
f_{\text{ave}} = \frac{\int_a^b f(x) \, dx}{b - a}.
\]

Here \( b - a \) is the length of \( I \), the measure of how big the interval of integration is.

• Similarly, in Calculus 3, the average value of \( f(x, y) \) over the region \( D \) in the \( xy \)-plane is defined by

\[
f_{\text{ave}} = \frac{\iint_D f(x, y) \, dA}{\text{area of } D} = \frac{\iint_D f(x, y) \, dA}{\iint_D 1 \, dA}.
\]

Here, the area of \( D \) is \( \iint_D 1 \, dA \), the measure of how big the region \( D \) is, since

\[
\iint_D 1 \, dA = \lim_{\|P\| \to 0} \sum_{i=1}^m \sum_{j=1}^n 1 \Delta x_i \Delta y_j.
\]

• Analogously, the average value of \( f(x, y, z) \) over the solid \( E \) in \( xyz \)-space is defined by

\[
f_{\text{ave}} = \frac{\iiint_E f(x, y, z) \, dV}{\text{volume of } E} = \frac{\iiint_E f(x, y, z) \, dV}{\iiint_E 1 \, dV}.
\]

Here, the volume of \( E \) is \( \iiint_E 1 \, dV \), the measure of how big the solid \( E \) is, since

\[
\iiint_E 1 \, dV = \lim_{\|P\| \to 0} \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^q 1 \Delta x_i \Delta y_j \Delta z_k.
\]

• In summary, the average value is the integral of the function divided by the measure.

**Measure differentials**

Here are area and volume differentials in different coordinate systems that we’ll encounter in this chapter. They may be permuted.

- \( dA = dx \, dy = r \, dr \, d\theta = J \, du \, dv \)
- \( dV = dx \, dy \, dz = r \, dz \, dr \, d\theta = \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta = J \, du \, dv \, dw \)