1.
- The parameter value corresponding to $P(3,0,2)$ is $t = 1$.
- With $\mathbf{r}(t) = \left[1 + 2\sqrt{t}, t^3 - t, t^3 + t\right]$, we have $\mathbf{r}'(t) = \left[t^{-1/2}, 3t^2 - 1, 3t^2 + 1\right]$. Thus $\mathbf{v} = \mathbf{r}'(1) = [1, 2, 4]$.
- A parametric representation of the tangent line is $\mathbf{L}(u) = [x(u), y(u), z(u)] = \mathbf{P} + u\mathbf{v} = [u + 3, 2u, 4u + 2]$.
- Now $0 = z(u) = 4u + 2$ implies $u = -\frac{1}{2}$, whence $\mathbf{L}\left(-\frac{1}{2}\right) = \left[\frac{5}{2}, -1, 0\right]$ is the position vector of the point of intersection of the tangent line with the xy-plane.

2.
- Solve $\mathbf{0} = \overrightarrow{\nabla} f = \left[3x^2 - 8y, 81y^2 - 8x\right]$ to obtain critical points $(0,0)$ and $\left(\frac{8}{9}, \frac{8}{27}\right)$.
- The Hessian is $H = \begin{bmatrix} 6x & -8 \\ -8 & 162y \end{bmatrix}$ and the LPMDs are $\{6x, 972xy - 64\}$.
- At $(0,0)$, the LPMDs are $\{0, -64\}$, which indicates a saddle point. Also, $f(0,0) = 0$.
- At $\left(\frac{8}{9}, \frac{8}{27}\right)$, the LPMDs are $\left\{\frac{16}{3}, 192\right\}$, which signifies a local minimum. Also, $f\left(\frac{8}{9}, \frac{8}{27}\right) = -\frac{512}{729} \approx -0.70$.

3.
- Let x, y, z represent the length, width, and height of the aquarium. The volume is $xyz = 160$, whence $z = \frac{160}{xy}$.
- The cost is $C = 5xy + 1(2xz + 2yz) = 5xy + \frac{320}{x} + \frac{320}{y}$ after substituting for z.
- Solve $\mathbf{0} = \overrightarrow{\nabla} C = \left[5y - \frac{320}{x^2}, 5x - \frac{320}{y^2}\right]$ to obtain $x = y = 4$ ft. Then $z = 10$ ft.
- Since $x, y > 0$, as (x, y) approaches the coordinate axes (the boundary of the first quadrant), we have $C \to \infty$. Therefore, the minimum cost is $C(4,4) = \$240$.
- [Lagrange multipliers also work.]

4.
- When the cardioid intersects the line, we have $8 + 8\sin\theta = 6/\sin\theta$, whence $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$.
- The area is $A = \int_{\pi/6}^{5\pi/6} \int_{6/\sin\theta}^{8+8\sin\theta} 1\, r\, dr\, d\theta = 4\left(9\sqrt{3} + 8\pi\right) \approx 162.88\ \text{cm}^2$.

5.
- Solve $1 + \sqrt{x} = \frac{1}{3}x + 1$ to obtain $x = 0, 9$. Observe (from the picture or analytically) that the curve lies above the line.
- The mass is $m = \iint_D \delta\, dA = \int_0^9 \int_{x/3+1}^{1+\sqrt{x}} xy\, dy\, dx = \frac{1863}{40} \approx 46.58$.
- The center of mass is $[\bar{x}, \bar{y}] = \frac{1}{m} \iint_D \delta\, [x, y]\, dA = \frac{1}{1863/40} \int_0^9 \int_{x/3+1}^{1+\sqrt{x}} xy\, [x, y]\, dy\, dx = \left[\frac{837}{161}, \frac{482}{161}\right] \approx [5.20, 2.99]$.

6.
- The radius of the circular arc is 9. Parameterize said arc as $\mathbf{g}(t) = [9\cos t, 9\sin t]$, $\frac{5}{4}\pi \le t \le \frac{7}{4}\pi$.
- The mass is $\int_C \delta\, ds = \int_a^b \delta(\mathbf{g}(t)) \left\|\mathbf{g}'(t)\right\|\, dt = \star$.
- The composition is $\delta(\mathbf{g}(t)) = -9\sin t$.
- The vector derivative is $\mathbf{g}'(t) = [-9\sin t, 9\cos t]$.
- Its magnitude is $\left\|\mathbf{g}'(t)\right\| = 9$.
- Therefore, $\star = \int_{5\pi/4}^{7\pi/4} -81\sin t\, dt = 81\sqrt{2} \approx 114.55$ grams.

7.
- The work done is $W = \int_C \mathbf{w} \cdot d\mathbf{g} = \int_a^b \mathbf{w}(\mathbf{g}(t)) \cdot \mathbf{g}'(t)\, dt = \star$.
- The composition is $\mathbf{w}(\mathbf{g}(t)) = \left[e^t, \ln(t+1), \tan^{-1} t\right]$.
- The vector derivative is $\mathbf{g}'(t) = \left[3t^2, 2t, 1\right]$.
- The dot product is $\mathbf{w}(\mathbf{g}(t)) \cdot \mathbf{g}'(t) = 2t\ln(t+1) + 3t^2e^t + \tan^{-1} t$.
- Hence $\star = \int_0^1 2t\ln(t+1) + 3t^2e^t + \tan^{-1} t\, dt = 3e + \frac{1}{4}\pi - \frac{11}{2} - \frac{1}{2}\ln 2 \approx 3.09$ joules.

8.
- Recall that $z = f(x, y) = \sqrt{x + e^{4y}}$.
- An equation of the tangent plane is given by $z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$.
- Now $f_x = \frac{1}{2\sqrt{x+e^{4y}}}$, whereas $f_y = \frac{2e^{4y}}{\sqrt{x+e^{4y}}}$.
- Thus $z_0 = f(3, 0) = 2$, $f_x(3, 0) = \frac{1}{4}$, and $f_y(3, 0) = 1$.
- Therefore, an equation of the tangent plane is $z - 2 = \frac{1}{4}(x - 3) + 1(y - 0)$ or $z = \frac{1}{4}x + y + \frac{5}{4}$.

9.
- The boundary surface is *closed.* Apply the Divergence Theorem and save yourself a lot of work! We have $\iint_S \mathbf{w} \cdot d\mathbf{S} = \iiint_E \operatorname{div} \mathbf{w}\, dV = \star$.
- The shadow region in the xy-plane is the circular disk $x^2 + y^2 \le 16$, $z = 0$. Its radius is 4.
- Now $\operatorname{div} \mathbf{w} = 2$, so $\star = \int_0^{2\pi} \int_0^4 \int_{r^2}^{16} 2\, r\, dz\, dr\, d\theta = 256\pi \approx 804.25$.

10.
- By drawing a picture of the region of integration, we may express it as $0 \le y \le x^2$, $0 \le x \le 2$.
- Accordingly, the integral becomes $\int_0^2 \int_0^{x^2} \frac{y^2 e^{x^2}}{x}\, dy\, dx = \frac{1}{3}\left(5e^4 - 1\right) \approx 90.66$.

11. The volume is $V = \iiint_E 1\, dV = \int_0^{2\pi} \int_0^{\pi} \int_0^{\phi} 1\, \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta = \frac{2}{3}\pi^2\left(\pi^2 - 6\right) \approx 25.46\ \text{cm}^3$.