1. Which is an equation of the sphere with center \((-6, -1, 2)\) and radius \(2\sqrt{3}\)?

(a) \(x^2 + y^2 + z^2 - 12x - 2y + 3z = 24\sqrt{3} - 41\)
(b) \(x^2 + y^2 + z^2 = 6x + 4y + 10z\)
(c) \(x^2 + 12x + y^2 + 2y + z^2 - 4z = -29\)
(d) \(x^2 - 6x + y^2 - 4y + z^2 + 3z = 8 (3)^{3/2}\)

(c) Said sphere has equation \((x + 6)^2 + (y + 1)^2 + (z - 2)^2 = (2\sqrt{3})^2\). Expanding and moving constants to the right-hand side yields \(x^2 + 12x + y^2 + 2y + z^2 - 4z = -29\).

2. Which of the following is a unit vector parallel to \([-2, 4, 3]\)?

(a) \([- \frac{2}{3}, \frac{4}{3}, 1]\)
(b) \([- \frac{2}{\sqrt{5}}, \frac{4}{5\sqrt{5}}, \frac{3}{\sqrt{5}}]\)
(c) \([-1, 2, \frac{3}{2}]\)
(d) \([\frac{2}{29\sqrt{29}}, - \frac{4}{29\sqrt{29}}, - \frac{3}{29\sqrt{29}}]\)

(d) A unit vector parallel to \(v = [-2, 4, 3]\) is \(\hat{v} = \frac{v}{\|v\|} = \frac{[-2, 4, 3]}{\sqrt{4 + 16 + 9}} = \left[\frac{-2}{\sqrt{29}}, \frac{4}{\sqrt{29}}, \frac{3}{\sqrt{29}}\right]\). Note that \(-\hat{v}\), which is equivalent to the last choice, is also a unit vector parallel to \(v\).

3. A bicycle pedal is pushed by a foot with a 60 N force as shown in the diagram below. The shaft is 18 cm long. Find the magnitude of the torque about \(P\).

(a) 7.70 joules (b) 10.64 joules (c) 7.30 joules (d) 10.80 joules

(b) The magnitude of the torque is \(\|\vec{r} \times \vec{F}\| = \|\vec{r}\| \|\vec{F}\| |\sin \theta| = (60) (0.18) (\sin 100^\circ) \approx 10.64 \text{ J.}\)

[Please turn the page over for the last problem.]
4. Two lines have the following parametric representations.

\[ L_1(s) = [2s + 4, 4s - 5, -3s + 1] \quad L_2(t) = [t + 2, 3t - 1, 2t] \]

Are the lines parallel, skew, and/or intersecting? In your work, if they intersect, show the point of intersection.

(a) parallel (b) skew (c) intersect (d) both (a) and (b)

- Direction vectors for the lines are \( \mathbf{v} = [2, 4, -3] \) and \( \mathbf{w} = [1, 3, 2] \), respectively. Since neither is a scalar multiple of the other, these vectors (and hence their corresponding lines) are not parallel.

- If the lines intersect, we have \( L_1(s) = L_2(t) \). This implies

\[
\begin{align*}
2s + 4 &= t + 2 \\
4s - 5 &= 3t - 1 \\
-3s + 1 &= 2t
\end{align*}
\]

- The first equation gives \( t = 2s + 2 \). Substituting this into the second equation yields \( 4s - 5 = 3(2s + 2) - 1 \), whence \( 2s = -10 \) or \( s = -5 \) and thus \( t = 2(-5) + 2 = -8 \).

- However, if we then substitute \( s = -5 \) and \( t = -8 \) into the third equation, we obtain \( 16 = -16 \), which is a contradiction! So there are no pair of values for \( s \) and \( t \) that make all three equations true simultaneously. Hence the lines do not intersect.

- (b) We conclude that the lines are skew (since they are not parallel and nonintersecting).