1. Compute the gradient vector field of $f(x, y) = xy^2$.

- The gradient vector field is $\nabla f = [f_x, f_y] = [y^2, 2xy]$, a picture of which appears below.

2. Evaluate $\int_C x^2 \, ds$ where $C$ is the unit circle $x^2 + y^2 = 1$ traversed counterclockwise once.

- This is a line integral of a scalar field along a curve with respect to arc length. Here is our blueprint.

3. Calculate $\int_C \mathbf{w} \cdot d\mathbf{g}$ where $\mathbf{w} = \left[ e^{x^{-1}}, xy, \frac{z}{xy} \right]$ and $\mathbf{g} = [x(t), y(t), z(t)] = [t^2, t^3, t^4]$. 

- This is a line integral of a vector field along a curve. Here is our blueprint.
4. Use the inverse transformation $T^{-1}: u = y + x, v = y - x$ [and its corresponding forward transformation $T: x = x(u, v), y = y(u, v)$ which you must determine] to evaluate the integral $\int \int_R \frac{\cos(y - x)}{\cos(y + x)}\,dA$.

Here $R$ is the slanted trapezoid with corners $(1, 0), \left(\frac{1}{2}, 0\right), (0, \frac{1}{2})$, and $(0, 1)$.

- Here is a picture of the region of integration in the $xy$-plane.

- The slanted boundary lines in the picture are $y = \frac{1}{2} - x$ and $y = 1 - x$. These correspond to $u = y + x = \frac{1}{2}$ and $u = y + x = 1$, respectively in the $uv$-plane.

- Solving the inverse transformation for $x$ and $y$ yields the forward transformation, $T: x = \frac{1}{2}(u - v), y = \frac{1}{2}(u + v)$.

- The $x$-axis boundary is $0 = y = \frac{1}{2}(u + v)$, whence $v = -u$ in the $uv$-plane.

- The $y$-axis boundary is $0 = x = \frac{1}{2}(u - v)$, whence $v = u$ in the $uv$-plane.

- The Jacobian matrix is $JM = \begin{bmatrix} x_u & x_v \\ y_u & y_v \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{bmatrix}$. Its determinant is $JD = \frac{1}{4} - (-\frac{1}{4}) = \frac{1}{2}$.

- The Jacobian factor is $J = JF = |JD| = \frac{1}{2}$.

- (b) Accordingly, the transformed integral is

$$\int \int_R \frac{\cos(y - x)}{\cos(y + x)}\,dA = \int \int_{D_{uv}} f(x(u, v), y(u, v))\,J\,du\,dv = \int_{1/2}^1 \int_{-u}^u \cos v \left(\frac{1}{2}\right)\,dv\,du = \ln \left(\frac{\cos \frac{1}{2}}{\cos 1}\right) \approx 0.49.$$