

Spring 2004 Math 253/501–503

13 Multiple Integrals

Parametric Boundary Curve Drawings

Wed, 24/Mar ©2004, Art Belmonte

Summary

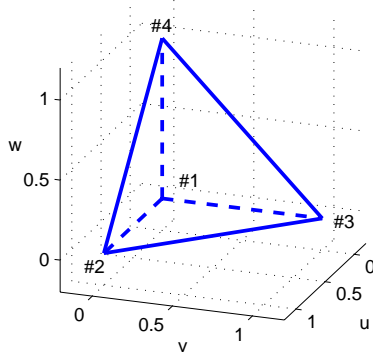
One way to depict a solid in 3-D space is with a *parametric boundary curve drawing* (PBCD). You basically draw the *edges* of the solid where its faces (surface patches) intersect.

MATLAB Examples

864/76ga

Make a PBCD of the tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$ in uvw -space. Use **view(110,20)** for your viewpoint.

Stewart 864/76a: Tetrahedron in uvw -space



Solution

This is a relatively easy problem since the edges are all straight line segments, each of which just requires a starting point and ending point to draw.

We did this one in MATLAB at the end of lecture on Tue, 23/Mar.

1. You may wish to draw a rough sketch on paper and label your vertices as #1, #2, #3, #4.
2. Specify the uvw -triples as rows of a matrix **M**.
3. Extract entire collections of u -, v -, and w -coordinates using MATLAB indexing.
4. Stroke the line segments with **plot3** (since you're in space). Do these one at a time using pairs of start and end points.

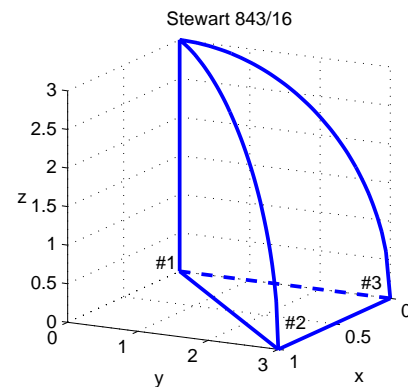
5. Season to taste; i.e., adjust line thickness, style, etc. For example, indicate hidden lines by using a dashed style.

Here is a script M-file; tedious, but that's life...

```
%
delete s864x76ga.txt; diary s864x76ga.txt
clear; clc; close all; echo on
%
% Stewart 864/76ga
%
M = [0 0 0; 1 0 0; 0 1 0; 0 0 1]; % Hunk 'o' data
u = M(:,1); v = M(:,2); w = M(:,3); % Extract columns.
% "Hidden" lines
plot3(u([1,2]), v([1,2]), w([1,2]), '--', 'LineWidth', 2)
hold on
plot3(u([1,3]), v([1,3]), w([1,3]), '--', 'LineWidth', 2)
plot3(u([1,4]), v([1,4]), w([1,4]), '--', 'LineWidth', 2)
% Regular lines
plot3(u([2,4]), v([2,4]), w([2,4]), 'LineWidth', 2)
plot3(u([3,4]), v([3,4]), w([3,4]), 'LineWidth', 2)
plot3(u([2,3]), v([2,3]), w([2,3]), 'LineWidth', 2)
% Viewpoint, etc.
view(110,20)
axis equal
axis([-0.2 1.2 -0.2 1.2 -0.2 1.2])
grid on
%
echo off; diary off
```

843/16g

Make a PBCD of the solid bounded by the cylinder $y^2 + z^2 = 9$ and the planes $x = 0$, $y = 3x$, and $z = 0$ in the first octant.



Solution

This one is more difficult since some intersections of surfaces are genuine curves, not just straight lines. These will need to be parameterized as in Section 11.6, $q.v$.

The straight line segments can be dealt with as before (just specify a segment's starting and ending points). Or you may plot them parametrically as a curve. In the code below, you'll see both.

Here are rectangular parameterizations of the two genuine curves. (You could alternatively use polar/cylindrical parameterizations in this instance.) Then finish it off as before.

1. The plane $x = 0$ intersects the cylinder $y^2 + z^2 = 9$ in the parametric curve $\mathbf{g}_1(t) = [0, t, \sqrt{9 - t^2}]$, $0 \leq t \leq 3$.
2. The plane $y = 3x$ intersects the cylinder $y^2 + z^2 = 9$ in the parametric curve $\mathbf{g}_2(t) = [\frac{1}{3}t, t, \sqrt{9 - t^2}]$, $0 \leq t \leq 3$.
3. Stroke the line segments one at a time using pairs of start and end points, just as in the previous problem. (This is the preferred way.)
4. Or draw a line segment as a curve: $\mathbf{g}_3 = [0, 0, t]$, $0 \leq t \leq 3$.
5. Season to taste.

A script M-file follows. Make sure to choose enough t -values to make the curves look smooth. Here we use 50 points and the same viewpoint as in the first problem.

One last thing: *make your expressions array-smart!*

```
%
delete s843x16g.txt; diary s843x16g.txt
clear; clc; close all; echo on
%
% Stewart 843/16g
%
t = linspace(0, 3, 50);
x1 = 0*t; y1 = t; z1 = sqrt(9-t.^2); % HELLO! array-smart!
x2 = y1/3; y2 = y1; z2 = z1;
x3 = 0*t; y3 = 0*t; z3 = t;
% A = [0 0 0]; B = [1 3 0]; C = [0 3 0];
M = [0 0 0; 1 3 0; 0 3 0];
x = M(:,1); y = M(:,2); z = M(:,3);
plot3(x([1,2]), y([1,2]), z([1,2]), 'LineWidth', 2)
grid on; hold on
plot3(x([2,3]), y([2,3]), z([2,3]), 'LineWidth', 2)
plot3(x([3,1]), y([3,1]), z([3,1]), '--', 'LineWidth', 2)
plot3(x1, y1, z1, 'LineWidth', 2)
plot3(x2, y2, z2, 'LineWidth', 2)
plot3(x3, y3, z3, 'LineWidth', 2)
view(110,20)
%
echo off; diary off
```