Section 3.1

1. [144/8] Solve \( y'' - 2y' - 2y = 0 \) via \texttt{deSolve}, then semiautomatically.

2. [144/20] Solve \( 2y'' - 3y' + y = 0, \ y(0) = 2, \ y'(0) = \frac{1}{2} \), via \texttt{deSolve}, then semiautomatically. Determine the maximum value of the solution as well as the point when the solution is zero. Plot the solution.

Section 3.2

1. [155/6] Find the Wronskian matrix and determinant for the pair of functions \( \cos^2 \theta, 1 + \cos 2\theta \). Simplify the latter via \texttt{tCollect}, a trigonometric simplification command.

2. [155/9] Use the linear Existence and Uniqueness Theorem (EUT) to determine the largest interval for which the specified initial value problem (IVP) has a unique solution. (You don’t need to solve the IVP.)

\[
t(t - 4)y'' + 3ty' + 4y = 2, \quad y(3) = 0, \quad y'(3) = -1
\]

3. [156/18] If the Wronskian determinant of \( f \) and \( g \) is \( r^2 e^t \) and \( f(t) = t \), find \( g(t) \).

4. [156/24] Verify that \( y_1 = \cos 2t \) and \( y_2 = \sin 2t \) are solutions of \( y'' + 4y = 0 \). Do they constitute a fundamental set of solutions?

5. [156/26] Same drill as #4 for \( y_1 = x \) and \( y_2 = xe^x \) with \( x^2y'' - x(x + 2)y' + (x + 2)y = 0, \ x > 0 \).

Section 3.3

Do these semiautomatically. Check with \texttt{deSolve} as a first resort or else via \texttt{lldoeval} and substitution.

1. [164/16] Find the general solution of the differential equation \( y'' + 4y' + \frac{25}{4}y = 0 \).

2. [164/18] Solve the initial value problem

\[
y'' + 4y' + 5y = 0, \quad y(0) = 1, \quad y'(0) = 0.
\]

3. [166/36] Find the general solution of the Cauchy-Euler equation \( r^2y'' + 4ty' + 2y = 0, \ t > 0 \).

Section 3.4

Do these semiautomatically. Check with \texttt{deSolve} as a first resort or else via \texttt{lldoeval}, etc.

1. [173/8] Find the general solution of the differential equation \( 16y'' + 24y' + 9y = 0 \).

2. [174/24] Find the general solution of the Cauchy-Euler equation \( t^2y'' + 2ty' - 2y = 0, \ t > 0 \).

3. [175/44] Find the general solution of the Cauchy-Euler equation \( 4t^2y'' - 8ty' + 9y = 0, \ t > 0 \).

4. [166/42] Find the general solution of the Cauchy-Euler equation \( t^2y'' + 7ty' + 10y = 0, \ t > 0 \).

5. [174/27] Given that \( y_1 = \sin (x^2) \) is a solution of the differential equation \( xy'' - y' + 4x^3y = 0, \ x > 0 \), find a second linearly independent solution via reduction of order (use \texttt{Lroof}, “reduction of order formula”). Then give the general solution.

Section 3.5

Use MUC (Method of Undetermined Coefficients) to solve these problems semiautomatically. Check with \texttt{deSolve}, seasoned with \texttt{tCollect} and/or \texttt{expand} when needed!

1. [184/2] Find the general solution of the differential equation \( y'' + 2y' + 5y = 3\sin 2t \).

2. [184/4] Same as #1 for \( y'' + y' - 6y = 12e^{3t} + 12e^{-2t} \).

3. [184/6] Same as #1 for \( y'' + 2y' = 3 + 4\sin 2t \).

4. [184/18] Solve the initial value problem

\[
y'' - 2y' - 3y = 3te^{2t}, \quad y(0) = 1, \quad y'(0) = 0.
\]

5. [184/15-alt] Same as #1 for

\[
y'' + y' - 2y = 3t^2 - 4t + 8.
\]

Section 3.6

Use VOP (Variation of Parameters) to solve these problems semiautomatically. Check with \texttt{deSolve} as a first resort or else via \texttt{lldoeval}.

1. [190/6] Find a general solution for \( y'' + 9y = 9\sec^2 3t, \ 0 < t < \pi/6 \).

2. [190/10] Same as #1 for \( y'' - 2y' + y = \frac{e^t}{1 + t^2}, \ t \in \mathbb{R} \).
Section 3.7

1. [203/4] Given \( u = -2 \cos \pi t - 3 \sin \pi t \), determine the natural frequency \( \omega_0 \), amplitude \( R \), and phase \( \delta \) so as to write \( u \) in the form \( u = R \cos (\omega_0 t - \delta) \). In general, let \( u = A \cos (\omega_0 t) + B \sin (\omega_0 t) \). Then \( R = \sqrt{A^2 + B^2} \) and \( \tan \delta = B/A \), with \( \delta \) in the appropriate quadrant as determined by the signs of \( A \) and \( B \).

2. [203/7] A mass weighing 3 lb stretches a spring 3 inches or \( \frac{1}{4} \) ft. The mass is pushed upward, contracting the spring a distance of 1 inch or \( \frac{1}{12} \) ft, then set in motion with a downward velocity of 2 ft/s. If there is no damping, find the position \( u \) of the mass at any time \( t \). Also determine the frequency, period, amplitude, and phase of the motion.

3. [204/8] A series electric circuit has a capacitor of \( C = \frac{1}{2} \times 10^{-6} \) F (farads) and an inductor of \( L = 1 \) H (henries). If the initial charge on the capacitor is \( Q(0) = Q_0 = 10^{-5} \) C (coulombs), and no initial current \( Q'(0) = I(0) = I_0 = 0 \) A (amperes), find the charge \( Q \) on the capacitor at any time \( t \).

4. [204/11] A spring is stretched 10 cm = \( \frac{1}{10} \) m by a force of 3 N. A mass of 2 kg is hung from the spring and is also attached to a viscous damper that exerts a force of 3 N when the velocity of the mass is 5 m/s. If the mass is pulled down 5 cm = \( \frac{1}{20} \) m below its equilibrium position and given an initial downward velocity of 10 cm/s = \( \frac{1}{10} \) m/s, determine its position \( u \) at any time \( t \). Find the quasi frequency \( \mu \) as well as the ratio \( \mu / \omega_0 \) of \( \mu \) to the natural frequency \( \omega_0 \) of the corresponding undamped motion.

5. [206/29] The position of a certain spring-mass system satisfies the initial value problem \( u'' + \frac{1}{4} u' + 2u = 0 \), \( u(0) = 0 \), \( u'(0) = 2 \).
   
   (a) Solve the initial value problem.
   
   (b) Plot \( u \) versus \( t \) and \( u' \) versus \( t \) on the same axes.
   
   (c) Parametrically plot \( u' \) vs \( u \) in the phase plane.

Section 3.8

1. [217/4] Write \( \sin 3t + \sin 4t \) as a product of two trigonometric functions of different frequencies via a trigonometric identity.

2. [217/8] A mass of 5 kg stretches a spring 10 cm or \( \frac{1}{10} \) m. The mass is acted on by an external force of \( F(t) = 10 \sin (t/2) \) N and moves in a medium that imparts a viscous force of 2 N when the speed of the mass is 4 cm/s = \( \frac{1}{10} \) m/s. The mass is set in motion from its equilibrium position with an initial velocity of 3 cm/s = \( \frac{3}{100} \) m/s.
   
   (a) Find the position \( u \) of the mass at any time \( t \).
   
   (b) Identify the transient and steady state parts of the solution.
   
   (c) Plot the graph of the solution.

3. [217/10] A mass that weighs 8 lb stretches a spring 6 inches or \( \frac{1}{2} \) ft. The system is acted upon by an external force of \( F(t) = 8 \sin 8t \) lb. The mass is pulled down 3 inches or \( \frac{1}{4} \) ft and then released.
   
   (a) Determine the position \( u \) at any time \( t \).
   
   (b) Determine the first four times at which the velocity of the mass is zero.
   
   (c) Note the phenomenon exhibited.

4. [218/18] Consider the forced but undamped system described by the initial value problem
   \[ u'' + u = 3 \cos \omega t, \quad u(0) = 0, \quad u'(0) = 0. \]
   
   (a) Find the solution for \( \omega \neq 1 \).
   
   (b) The natural frequency of the unforced system is \( \omega_0 = 1 \). Show this.
   
   (c) Plot the solution \( u \) versus \( t \) for \( \omega = \frac{9}{10} \).
   
   (d) Note the phenomenon exhibited.