Fall 2003 Math 308/501–502
9 Matrix Methods for Linear Systems
9.7M Nonhomogeneous Lin Sys: Method of Undetermined Coefficients
Mon, 24/Nov ©2003, Art Belmonte

Summary

Method of Undetermined Coefficients for Systems

Let $\mathbf{A}$ be an $n \times n$ real *constant* matrix and $\mathbf{f}$ an $n \times 1$ column vector whose elements are sums of products of real polynomials, sines, cosines, and/or exponentials involving the independent variable t. We may use the method of undetermined coefficients procedure from Section 6.3 as a *guide* to finding a particular solution $\mathbf{x}_p$ to the nonhomogeneous linear system $\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{f}$ or $L[\mathbf{x}] = \mathbf{x}' - \mathbf{A}\mathbf{x} = \mathbf{f}$. The undetermined coefficients involved are now symbolic *vector* constants.

Moreover, in case an element of $\mathbf{f}$ is replicated in a general solution of the associated homogeneous linear system $L[\mathbf{x}] = \mathbf{0}$, the original choice for a particular solution must *not only* be multiplied by the smallest positive integer power of t so that no term of the particular solution $\mathbf{x}_p$ is a solution of the homogeneous equation $L[\mathbf{x}] = \mathbf{0}$, *but also* by all lower nonnegative integer powers of t as well. This is easier said than done. Indeed, when this level of complexity is reached, it is simpler to resort to **variation of parameters**, the other technique for finding particular solutions that we encountered. This will be discussed later in lecture handout **9.7V**.

Superposition Principle

For $k = 1, 2, \ldots, M$, let $\mathbf{x}_{p_k}$ be a solution of $L[\mathbf{x}] = \mathbf{f}_k$. Then for any constants $c_1, \ldots, c_M$, the function $\mathbf{x}_p = \sum_{k=1}^{M} c_k \mathbf{x}_{p_k}$ solves the nonhomogeneous linear system $L[y] = \sum_{k=1}^{M} c_k f_k$. (This follows immediately from the fact that L is a linear operator.)

Hand Examples

In our first example, we'll do things soup-to-nuts by hand. In the next example, we'll assume we have the necessary eigenpairs (i.e., pairs of eigenvalues with associated eigenvectors) so as to rapidly form a general solution of the associated homogeneous system. Then we'll proceed to the main course: finding a particular solution of the nonhomogeneous system.

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Find a general solution to the nonhomogeneous system $\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{f}$, where $\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$ and $\mathbf{f} = \begin{bmatrix} -t-1 \\ -4t-2 \end{bmatrix}$.

Solution

Here is our overall solution strategy.

1. Find a general solution $\mathbf{x}_h$ to the associated homogeneous system $\mathbf{x}' = \mathbf{A}\mathbf{x}$.
2. Find a particular solution $\mathbf{x}_p$ to the nonhomogeneous system $\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{f}$.
3. Form a general solution of the nonhomogeneous system: $\mathbf{x} = \mathbf{x}_p + \mathbf{x}_h$.

Along the way, we'll flesh out details. Let's get the party started.

1. **Computation of $\mathbf{x}_h$.**
   (a) **Eigenvalues of A.** Solve $\det(\mathbf{A} - r\mathbf{I}) = 0$.
   $\begin{vmatrix} 1-r & 1 \\ 4 & 1-r \end{vmatrix} = r^2 - 2r - 3 = (r+1)(r-3) = 0$, whence $r = -1, 3$.
   (b) **Associated Eigenvectors.** Find a nonzero vector in the nullspace of the RREF of $\mathbf{A} - r\mathbf{I}$.
   - For $r = -1$, $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \overset{\text{rref}}{\rightarrow} \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 0 \end{bmatrix} \Rightarrow$ eigenpair $-1 \leftrightarrow \begin{bmatrix} -1 \\ 2 \end{bmatrix}$.
   - For $r = 3$, $\begin{bmatrix} -2 & 1 \\ 4 & -2 \end{bmatrix} \overset{\text{rref}}{\rightarrow} \begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & 0 \end{bmatrix} \Rightarrow$ eigenpair $3 \leftrightarrow \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

   (c) **Form $\mathbf{x}_h$.** A general solution of $\mathbf{x}' = \mathbf{A}\mathbf{x}$ is
   $$\mathbf{x}_h = c_1 e^{-t} \begin{bmatrix} -1 \\ 2 \end{bmatrix} + c_2 e^{3t} \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$
2. **Computation of $\mathbf{x}_p$.**
   (a) Rewrite $\mathbf{f} = \begin{bmatrix} -t-1 \\ -4t-2 \end{bmatrix}$ as $\mathbf{f} = t\mathbf{k}_1 + \mathbf{k}_2$, where $\mathbf{k}_1 = \begin{bmatrix} -1 \\ -4 \end{bmatrix}$ and $\mathbf{k}_2 = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$. Note that no element of $\mathbf{f}$ occurs in the same element of a solution of the homogeneous equation. There is no interference.
   (b) Accordingly, the form of $\mathbf{x}_p$ is $\mathbf{x}_p = t\mathbf{a} + \mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are undetermined *vector* constants.

(c) Substitute $\mathbf{x} = \mathbf{x}_p$ into $\mathbf{x}' - \mathbf{A}\mathbf{x} - \mathbf{f} = \mathbf{0}$ and collect like terms.

$$\begin{aligned} \mathbf{a} - \mathbf{A}(t\mathbf{a} + \mathbf{b}) - t\mathbf{k}_1 - \mathbf{k}_2 &= \mathbf{0} \\ -\mathbf{a} + \mathbf{A}(t\mathbf{a} + \mathbf{b}) + t\mathbf{k}_1 + \mathbf{k}_2 &= \mathbf{0} \\ t(\mathbf{A}\mathbf{a} + \mathbf{k}_1) + (\mathbf{A}\mathbf{b} + \mathbf{k}_2 - \mathbf{a}) &= \mathbf{0}, \text{ for all } t \in \mathbb{R}. \end{aligned}$$

Thus $\mathbf{A}\mathbf{a} + \mathbf{k}_1 = \mathbf{0}$ implies $\mathbf{a} = -\mathbf{A}^{-1}\mathbf{k}_1 = -\mathbf{A}\backslash\mathbf{k}_1$. Similarly, $\mathbf{A}\mathbf{b} + \mathbf{k}_2 - \mathbf{a} = \mathbf{0}$ yields $\mathbf{b} = \mathbf{A}\backslash(\mathbf{a} - \mathbf{k}_2)$. Via MATLAB, we have $\mathbf{a} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$. Hence $\mathbf{x}_p = t\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix}$.

3. **Computation of a general solution $\mathbf{x} = \mathbf{x}_p + \mathbf{x}_h$.**

$$\mathbf{x} = t\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} + c_1 e^{-t}\begin{bmatrix} -1 \\ 2 \end{bmatrix} + c_2 e^{3t}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

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Find a general solution to the nonhomogeneous system $\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{f}$, where $\mathbf{A} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ and $\mathbf{f} = \begin{bmatrix} -4\cos t \\ -\sin t \end{bmatrix}$.

Solution

1. MATLAB yields eigenpairs $0 \leftrightarrow \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $4 \leftrightarrow \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, whence $\mathbf{x}_h = c_1\begin{bmatrix} 1 \\ -1 \end{bmatrix} + c_2 e^{4t}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

2. Rewrite $\mathbf{f}$ as $\mathbf{f} = \cos t\,\mathbf{k}_1 + \sin t\,\mathbf{k}_2$, where $\mathbf{k}_1 = \begin{bmatrix} -4 \\ 0 \end{bmatrix}$ and $\mathbf{k}_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$. Again, there is no interference with the homogeneous general solution. Put $\mathbf{x}_p = \cos t\,\mathbf{a} + \sin t\,\mathbf{b}$ into $\mathbf{A}\mathbf{x} + \mathbf{f} - \mathbf{x}' = \mathbf{0}$.

$$\begin{aligned} \mathbf{A}(\cos t\,\mathbf{a} + \sin t\,\mathbf{b}) + \cos t\,\mathbf{k}_1 + \sin t\,\mathbf{k}_2 - (-\sin t\,\mathbf{a} + \cos t\,\mathbf{b}) &= \mathbf{0} \\ \cos t\,(\mathbf{A}\mathbf{a} + \mathbf{k}_1 - \mathbf{b}) + \sin t\,(\mathbf{A}\mathbf{b} + \mathbf{k}_2 + \mathbf{a}) &= \mathbf{0} \end{aligned}$$

In order for this last equation to hold for all t, we must have $\mathbf{A}\mathbf{a} + \mathbf{k}_1 - \mathbf{b} = \mathbf{0}$ and $\mathbf{A}\mathbf{b} + \mathbf{k}_2 + \mathbf{a} = \mathbf{0}$. This first equation yields $\mathbf{b} = \mathbf{A}\mathbf{a} + \mathbf{k}_1$. Substituting into the ensuing equation, we have

$$\begin{aligned} \mathbf{A}(\mathbf{A}\mathbf{a} + \mathbf{k}_1) + \mathbf{k}_2 + \mathbf{a} &= \mathbf{0} \\ (\mathbf{A}^2 + \mathbf{I})\mathbf{a} &= -(\mathbf{A}\mathbf{k}_1 + \mathbf{k}_2) \\ \mathbf{a} &= -\left(\mathbf{A}^2 + \mathbf{I}\right)^{-1}(\mathbf{A}\mathbf{k}_1 + \mathbf{k}_2) \\ \mathbf{a} &= -\left(\mathbf{A}^2 + \mathbf{I}\right)\backslash(\mathbf{A}\mathbf{k}_1 + \mathbf{k}_2) \end{aligned}$$

Via MATLAB, $\mathbf{a} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Thus $\mathbf{b} = \mathbf{A}\mathbf{a} + \mathbf{k}_1 = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$. Hence $\mathbf{x}_p = \cos t\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \sin t\begin{bmatrix} -2 \\ 2 \end{bmatrix}$.

3. A general solution of the nonhomogeneous system is $\mathbf{x} = \cos t\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \sin t\begin{bmatrix} -2 \\ 2 \end{bmatrix} + c_1\begin{bmatrix} 1 \\ -1 \end{bmatrix} + c_2 e^{4t}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

MATLAB Examples

555/2 [revisited]

Find a general solution to the nonhomogeneous system $\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{f}$, where $\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$ and $\mathbf{f} = \begin{bmatrix} -t - 1 \\ -4t - 2 \end{bmatrix}$.

Solution

Here we play fast and loose with symbols then creatively interpret "solutions" for **a** and **b**. We're pushing MATLAB beyond what it was intended to do. . . The diary file is rather long, so let's first look at the script M-file driver.

```
%
% NSS4-555/2
%
syms a b c1 c2 k1 k2 t
x = sym('x(t)');
A = sym([1 1; 4 1])
% Homogeneous general solution
[V,D] = eig(A)
x1 = exp(-t) * [-1; 2];
x2 = exp(3*t) * [1; 2];
X = [x1, x2]
c = [c1; c2]
xh = X*c; pretty(xh)
% Nonhomogeneous particular solution
syms A % (Temporarily make A a symbol.)
de0 = diff(x,t) - A*x - (t*k1 + k2)
xp = t*a + b;
eq0 = subs(de0, x, xp);
eq0 = collect(eq0, t)
[a b] = solve(-A*a-k1, a-A*b-k2, a, b);
pretty(a), pretty(b)
% Nitty gritty
A = sym([1 1; 4 1])
k1 = [-1; -4]
k2 = [-1; -2]
a = -A\k1
b = -A^(-2) * (A*k2 + k1)
xp = subs(xp)
% Nonhomogeneous general solution
x = xp + xh; pretty(x)
% Check
check = diff(x,t) - A*x - (t*k1 + k2)
%
echo off; diary off
```

And here is the diary file with input and output interspersed.

```
%
% NSS4-555/2
%
syms a b c1 c2 k1 k2 t
x = sym('x(t)');
A = sym([1 1; 4 1])

A =

[ 1, 1]
[ 4, 1]
```

```
% Homogeneous general solution
[V,D] = eig(A)

V =

[  1,  1]
[ -2,  2]


D =

[ -1,  0]
[  0,  3]

x1 = exp(-t) * [-1; 2];
x2 = exp(3*t) * [1; 2];
X = [x1, x2]

X =

[   -exp(-t),   exp(3*t)]
[  2*exp(-t), 2*exp(3*t)]

c = [c1; c2]

c =

[ c1]
[ c2]

xh = X*c; pretty(xh)

                        [ -exp(-t) c1 + exp(3 t) c2  ]
                        [                            ]
                        [2 exp(-t) c1 + 2 exp(3 t) c2]
% Nonhomogeneous particular solution
syms A % (Temporarily make A a symbol.)
de0 = diff(x,t) - A*x - (t*k1 + k2)
de0 =

diff(x(t),t)-A*x(t)-t*k1-k2

xp = t*a + b;
eq0 = subs(de0, x, xp);
eq0 = collect(eq0, t)

eq0 =

(-A*a-k1)*t+a-A*b-k2

[a b] = solve(-A*a-k1, a-A*b-k2, a, b);
pretty(a), pretty(b)

                                    k1
                                  - ----
                                     A

                                  A k2 + k1
                                - ---------
                                      2
                                     A
% Nitty gritty
A = sym([1 1; 4 1])

A =

[ 1, 1]
[ 4, 1]

k1 = [-1; -4]
k1 =
    -1
    -4
k2 = [-1; -2]
k2 =
    -1
    -2
a = -A\k1

a =

[ 1]
[ 0]

b = -A^(-2) * (A*k2 + k1)

b =

[ 0]
[ 2]

xp = subs(xp)

xp =

[ t]
[ 2]

% Nonhomogeneous general solution
x = xp + xh; pretty(x)

                      [  t - exp(-t) c1 + exp(3 t) c2  ]
                      [                                ]
                      [2 + 2 exp(-t) c1 + 2 exp(3 t) c2]
% Check
check = diff(x,t) - A*x - (t*k1 + k2)

check =

[ 0]
[ 0]

%
echo off; diary off
```

555/4 [revisited]

Find a general solution to the nonhomogeneous system $\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{f}$, where $\mathbf{A} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ and $\mathbf{f} = \begin{bmatrix} -4\cos t \\ -\sin t \end{bmatrix}$.

Solution

First the script M-file driver.

```
%
% NSS4-555/4
%
syms A a b c1 c2 k1 k2 t
x = sym('x(t)');
A = sym([2 2; 2 2])
% Homogeneous general solution
[V,D] = eig(A)
x1 = exp(0*t) * [-1; 1];
x2 = exp(4*t) * [1;1];
X = [x1, x2]
c = [c1; c2]
xh = X*c; pretty(xh)
% Nonhomogeneous particular solution
syms A % (Temporarily make A a symbol.)
de0 = diff(x,t) - A*x - (cos(t)*k1 + sin(t)*k2)
xp = cos(t)*a + sin(t)*b;
eq0 = subs(de0, x, xp);
eq0 = collect(eq0, cos(t));
eq0 = collect(eq0, sin(t))
[a b] = solve(-a-k2-A*b, -A*a-k1+b, a, b);
pretty(a), pretty(b)
% Nitty gritty
A = sym([2 2; 2 2])
I = sym(eye(2))
k1 = [-4; 0]
k2 = [0; -1]
a = - (I + A^2) \ (k2 + A*k1)
```

```
b = - (I + A^2) \ (A*k2 - k1)
xp = subs(xp)
% Nonhomogeneous general solution
x = xp + xh; pretty(x)
% Check
check = diff(x,t) - A*x - (cos(t)*k1 + sin(t)*k2)
%
%
echo off; diary off
```

And now the diary file.

```
%
% NSS4-555/4
%
syms A a b c1 c2 k1 k2 t
x = sym('x(t)');
A = sym([2 2; 2 2])

A =

[ 2, 2]
[ 2, 2]

% Homogeneous general solution
[V,D] = eig(A)

V =

[ -1,  1]
[  1,  1]


D =

[ 0, 0]
[ 0, 4]

x1 = exp(0*t) * [-1; 1];
x2 = exp(4*t) * [1;1];
X = [x1, x2]

X =

[       -1, exp(4*t)]
[        1, exp(4*t)]

c = [c1; c2]

c =

[ c1]
[ c2]

xh = X*c; pretty(xh)

                              [-c1 + exp(4 t) c2]
                              [                 ]
                              [c1 + exp(4 t) c2 ]
% Nonhomogeneous particular solution
syms A % (Temporarily make A a symbol.)
de0 = diff(x,t) - A*x - (cos(t)*k1 + sin(t)*k2)

de0 =

diff(x(t),t)-A*x(t)-cos(t)*k1-sin(t)*k2

xp = cos(t)*a + sin(t)*b;
eq0 = subs(de0, x, xp);
eq0 = collect(eq0, cos(t));
eq0 = collect(eq0, sin(t))

eq0 =

(-a-k2-A*b)*sin(t)+(-A*a-k1+b)*cos(t)

[a b] = solve(-a-k2-A*b, -A*a-k1+b, a, b);
pretty(a), pretty(b)

                     k2 + A k1
                   - ---------
                          2
                     1 + A

                     A k2 - k1
                   - ---------
                          2
                     1 + A
% Nitty gritty
A = sym([2 2; 2 2])

A =

[ 2, 2]
[ 2, 2]

I = sym(eye(2))

I =

[ 1, 0]
[ 0, 1]

k1 = [-4; 0]
k1 =
    -4
     0
k2 = [0; -1]
k2 =
     0
    -1
a = - (I + A^2) \ (k2 + A*k1)

a =

[ 0]
[ 1]

b = - (I + A^2) \ (A*k2 - k1)

b =

[ -2]
[  2]

xp = subs(xp)

xp =

[       -2*sin(t)]
[ cos(t)+2*sin(t)]

% Nonhomogeneous general solution
x = xp + xh; pretty(x)

                    [    -2 sin(t) - c1 + exp(4 t) c2    ]
                    [                                    ]
                    [cos(t) + 2 sin(t) + c1 + exp(4 t) c2]
% Check
check = diff(x,t) - A*x - (cos(t)*k1 + sin(t)*k2)

check =

[ 0]
[ 0]

%
%
echo off; diary off
```

555/26a

Find a general solution to the nonhomogeneous system $\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{f}$, where $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$ and $\mathbf{f} = \begin{bmatrix} 3e^t \\ 6e^t \end{bmatrix}$.

Solution

We readily construct a general solution to the associated homogeneous system with MATLAB.

$$\mathbf{x}_h = c_1 e^t \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

Notice that components of $\mathbf{f}$ can occur as components of $\mathbf{x}_h$. (For example, take $c_1 = 3$ and $c_2 = 0$. Then $3e^t$ occurs in the first element of $\mathbf{x}_h$.) Accordingly, we must adjust our first choice for $\mathbf{x}_p$ in the manner mentioned in the summary. Moreover, the fast-and-loose way we approached the first two problems won't work. (There's not enough structure inherent in our symbols.) Accordingly, we resort to the Full Monty: expressing all elements of vectors individually.

Therefore, let $\mathbf{x}_p = te^t\mathbf{a} + e^t\mathbf{b} = te^t \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + e^t \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$. This will work, but notice that there is not a unique solution to the linear system that we must solve after substituting into $\mathbf{x}' - \mathbf{A}\mathbf{x} - \mathbf{f} = \mathbf{0}$. So we have some freedom as well as some ambiguity. This, together with the mathematical gymnastics involved in the method of undetermined coefficients, is why we'll turn to the much more satisfying **variation of parameters** techique in the next lecture!

For now, peruse the needful in script M-file and diary formats as we construct our particular solution $\mathbf{x}_p = e^t \begin{bmatrix} 3 \\ 0 \end{bmatrix}$, then our general solution of the nonhomogeneous system $\mathbf{x} = \mathbf{x}_p + \mathbf{x}_h$. (REMARK: The fact that **a** turned out to be the zero vector happens sometimes. Always make sure, however, to use the full form of the particular solution $\mathbf{x}_p$. For other choices of $\mathbf{f}$ it may well turn out that **a** is *not* the zero vector!)

```
%==========
delete p557x26a.txt; diary p557x26a.txt
clear; clc; close all; echo on
%
% NSS4-557/26a
%
syms a b a1 a2 b1 b2 c1 c2 k t
x = sym('x(t)');
A = sym([0 1; -2 3])
% Homogeneous general solution
[V,D] = eig(A)
x1 = exp(t) * [1; 1];
x2 = exp(2*t) * [1; 2];
X = [x1, x2]
c = [c1; c2]
xh = X*c; pretty(xh)
% Nonhomogeneous particular solution
a = [a1;a2]; b = [b1;b2]; k = [3;6];
xp = t*exp(t)*a + exp(t)*b;
eq0 = diff(xp,t) - A*xp - exp(t)*k;
% All terms in the foregoing had a factor of exp(t).
% Since the output was VERY LONG, I suppressed it
% and divided out the exponential in the next command.
eq0 = simple(eq0 / exp(t));
eq0 = collect(eq0, t)
[a1 a2 b1 b2] = solve( ...
    -a2+a1, a1-3+b1-b2, 2*a1-2*a2, a2-6-2*b2+2*b1, ...
    a1, a2, b1, b2)
b2 = 0
b1 = subs(b1)
xp = subs(xp); xp = collect(xp, exp(t))
% A gen soln to NH lin sys
x = xp + xh; pretty(x)
%
f = exp(t)*k;
check = simple(diff(x,t) - A*x - f)
%
echo off; diary off
%==========
```

```
%
% NSS4-557/26a
%
syms a b a1 a2 b1 b2 c1 c2 k t
x = sym('x(t)');
A = sym([0 1; -2 3])

A =

[  0,  1]
[ -2,  3]

% Homogeneous general solution
[V,D] = eig(A)

V =

[ 1, 1]
[ 1, 2]


D =

[ 1, 0]
[ 0, 2]

x1 = exp(t) * [1; 1];
x2 = exp(2*t) * [1; 2];
X = [x1, x2]

X =

[     exp(t),   exp(2*t)]
[     exp(t), 2*exp(2*t)]

c = [c1; c2]

c =

[ c1]
[ c2]

xh = X*c; pretty(xh)

                          [ exp(t) c1 + exp(2 t) c2 ]
                          [                         ]
                          [exp(t) c1 + 2 exp(2 t) c2]
% Nonhomogeneous particular solution
a = [a1;a2]; b = [b1;b2]; k = [3;6];
xp = t*exp(t)*a + exp(t)*b;
eq0 = diff(xp,t) - A*xp - exp(t)*k;
% All terms in the foregoing had a factor of exp(t).
% Since the output was VERY LONG, I suppressed it
% and divided out the exponential in the next command.
eq0 = simple(eq0 / exp(t));
eq0 = collect(eq0, t)

eq0 =

[        (-a2+a1)*t+a1-3+b1-b2]
[ (2*a1-2*a2)*t+a2-6-2*b2+2*b1]

[a1 a2 b1 b2] = solve( ...
    -a2+a1, a1-3+b1-b2, 2*a1-2*a2, a2-6-2*b2+2*b1, ...
    a1, a2, b1, b2)

a1 =

0
%
%
%
```

```
a2 =

0


b1 =

3+b2


b2 =

b2

b2 = 0
b2 =
     0
b1 = subs(b1)
b1 =
     3
xp = subs(xp); xp = collect(xp, exp(t))

xp =

[ 3*exp(t)]
[        0]

% A gen soln to NH lin sys
x = xp + xh; pretty(x)

                     [3 exp(t) + exp(t) c1 + exp(2 t) c2]
                     [                                  ]
                     [    exp(t) c1 + 2 exp(2 t) c2     ]
%
f = exp(t)*k;
check = simple(diff(x,t) - A*x - f)

check =

[ 0]
[ 0]

%

echo off; diary off
```