Index

\( \nabla \) (del operator), 529–530

A

Acceleration, 492
  normal component of, 517
tangential component of, 517
Addition
  of matrices, 29
  in \( \mathbb{R}^n \), 118
  of vectors, 119
Adjacency matrix, 54
Adjoint of a matrix, 103, 463
Aerospace, 193, 303
Ampère’s law, 766–767, 768
  static case, 766
  time-varying case, 767, 768
Angle between vectors, 41, 107, 211, 217, 423, 451
Angular momentum, 545
Angular speed, 436
Angular velocity vector, 523
Approximation of functions, 262–265
Archimedes’ principle, 756
Arc length function, 507, 519
Arc length parameter, 507–509
Area
  zero, 561
Area element
  Cartesian, 606, 612
  general, 612
  polar, 606, 612
Area of a surface, 714–717
Astroid, 521
Astronomy
  Ceres orbit of Gauss, 235
ATLAST, 784
Augmented matrix, 7
Automobile leasing, 325
Average value, 615–619
  of a function on a surface, 775
Aviation, 193

B

Backslash operator, 787
Back substitution, 5
Basis, 144
  change of, 150–159
  orthonormal, 255
Bernoulli, Johann, 496
Bézier curve, 541
Binormal spherical image, 544
Binormal vector, 109
Binormal vector, see vector(s), binormal, 514
Block multiplication, 70–73
Brahe, Tycho, 495
Budget hyperplane, 458
Buoyant force, 756

C

Cauchy–Stieltjes integral, 120
Cauchy–Riemann equations, 544
Cauchy–Schwarz inequality, 213, 248, 452
Cavalieri’s principle, 552
Center of mass
  continuous, 621–625
  in \( \mathbb{R} \), 622
  in \( \mathbb{R}^2 \), 622
  in \( \mathbb{R}^3 \), 624
  discrete, 619–621
  of a wire, 699
Centroid, 625
Change of variables
  in double integrals, 599, 612
  in triple integrals, 607, 613
Characteristic equation, 300
Characteristic polynomial, 300
Characteristic value(s), 299
Characteristic vector, 299
Charge density, 764
Charge distribution, 764
Chebyshev polynomials, 285
  of the second kind, 288
Chemical equations, 20
Cholesky decomposition, 383
Circular cone, 492
Circulation, 661, 726
Circulation density, 748
Closed box, 579
Closed rectangle, 552, 556
Closure properties, 119
Clothoid, 543
C^1, 338
Coded messages, 105–106
Coefficient matrix, 7
Cofactor, 91, 459
Cofactor expansion, 91
Column space, 160, 228
Column vector notation, 28
Column vector(s), 27, 454
Commodity bundle, 458
Communication networks, 54
Companion matrix, 310
Complex
  eigenvalues, 305, 314–315
  matrix, 340
Computer graphics, 190
Conduction current density, 767
Conductivity, 772
Conic sections, 366–371
Connected, 689
Conservative vector field, 689
Consistency Theorem, 34, 161
Consistent 2
Continuity
  piecewise, 560
  Continuity equation, 766, 770
  for current densities, 770
  in fluid dynamics, 770

D

Dangling Web page, 329
Darboux formulas, 523
Darboux rotation vector, 523
Data fitting, least squares, 238–241
Defective matrix, 324
Definite quadratic form, 373
INDEX

Del operator (\(\vec{\nabla}\)), 529–530

Derivative
- directional of a vector field, 538
- normal, see normal derivative, 701, 757, 770

Determinant(s), 88–112, 432, 458
- cofactor expansion, 91
- definition, 93
- and eigenvalues, 300
- of elementary matrices, 98
- and linear independence, 138
- of a product, 100
- properties, 462
- of a singular matrix, 98
- of the transpose, 93
- of a triangular matrix, 94

DFT, 267

Diagonal matrix, 64

Diagonalizable matrix, 321

Diagonalizing matrix, 321

Differential geometry, 504

Differentiating under the integral sign, 647

Digital imaging, 361

Dilation, 190

Dimension, 146
- of row space and column space, 163
- Dimension Theorem, 281

Direct sum, 230

Direction cosines, 428

Discrete Fourier transform, 265–267

Displacement current density, 767

Displacement vector, 407

Distance
- between parallel planes, 448
- between point and line, 447–448, 479
- between point and plane, 479
- between skew lines, 448–449
- in 2-space, 211
- in n-space, 217
- in a normed linear space, 251

Divergence, 530–531, 746
- in cylindrical coordinates, 535, 757
- in spherical coordinates, 537, 758

Divergence theorem, see Gauss’s theorem in the plane, 680

Dominant eigenvalue, 328

Dot product, 420–427, 438
- definition, 421
- properties, 421

Double integral, 558
- in polar coordinates, 602–604

E

Economic models, 21–23

Edges of a graph, 54

Eigenspace, 300

Eigenvalue(s), 296–395
- complex, 305
- definition, 299
- and determinants, 300
- product of, 306
- of similar matrices, 307
- and structures, 296, 302, 393
- sum of, 306

of a symmetric positive definite matrix, 374

Eigenvector, 299

Electric field
- of a continuous charge distribution, 764
- of a single point charge, 762

Electrical networks, 19

Electromotive force, 767

Elementary matrix, 58
- determinant of, 98
- inverse of, 60

Elementary region in space, 582–585
- definition, 582
- in the plane, 563–569
- definition, 563

Ellipsoid
- parametrized, 718, 775

Endowment vector, 458

Epicycles, 495

Epicycloid, 482, 685

Epitrochoid, 482

Equation of continuity, see continuity equation, 770

Equation of first variation, 529

Equipotential line, 526

Equipotential set, 526

Equipotential surface, 526

Equivalent systems, 3–5, 58

Euclidean length, 211

Euclidean n-space, 27

Evolute, 542

Extension (\(f^{\text{ext}}\)), 564, 585

F

Factor analysis, 223

Faraday’s law, 767–768

Fast Fourier Transform, 267–269

Fenchel’s theorem, 700

Field
- scalar, see scalar field, 524
- vector, see vector field, 523

Finite dimensional, 146

First variation equation of, see equation of first variation, 529

Flow line, 527

Flow of a vector field, 529

FLT axis system, 194

Flux, 680, 726

Flux density, 748

Force
- buoyant, 756
- Fourier coefficients, 264
- complex, 264
- Fourier matrix, 267

Free variables, 13

Frenet–Serret formulas, 519–521

Fresnel integrals, 543

Frobenius norm, 247

Frobenius theorem, 389

Frobenius, 776

Fubini’s theorem, 556, 561, 581

Full rank, 168

Functions
- average value, 616
- along a curve, 699
- extension of (\(f^{\text{ext}}\)), 564, 585
- linear, 457
- mean value of, 616
- potential, 525, 689

Fundamental subspaces, 227

Fundamental Subspaces Theorem, 228

G

Gauge freedom, 778

Gauss, Carl Friedrich, 234

Gauss–Jordan reduction, 17

Gaussian elimination, 13

Gaussian quadrature, 287

Gauss’s law, 762–764, 768
- differential form, 764
- integral form, 764

Gauss’s theorem, 681, 743–746
- proof of, 753–755

Google, 329

Gradient, 529–530
- in cylindrical coordinates, 535
- in spherical coordinates, 537

Gradient field, 525

Gram–Schmidt process, 271–280
- modified version, 279

Graphs, 54

Gravitational potential, 630

Green’s first formula, 760

Green’s first identity, 701

Green’s formulas, 760

Green’s second formula, 760

Green’s second identity, 701

Green’s theorem, 677, 684
- proof of, 681–684
- vector reformulation of, 679

Green’s third formula, 760

Gyration, radius of, 627, 699, 776

H

Harmonic function, 545, 701, 761, 770

Harmonic motion, 317

Head-to-tail addition, 406

Heat equation, 770, 771
- uniqueness of solutions to, 771

Heat flux density, 739, 770

Helicoid, 718

Helix, 492

Hermite polynomials, 285

Hermitian matrix, 341
- eigenvalues of, 341

Hessian, 377

Hilbert matrix, 481

Homogeneous coordinates, 192

Homogeneous system, 20
- nontrivial solution, 20

Hotelling, H., 363

Hyperboloid
- parametrized, 774

Hyperplane
- budget, 458
- Hypersphere, 475
- Hyperspherical coordinates, 474–475

Index
Hypersurface, 475
Hypocycloid, 482
Hypotrochoid, 482

I
Idempotent, 57, 308, 461
Identity matrix, 50
Image, 180
Improper integral, 647–648
Inconsistent, 2
Indefinite quadratic form, 373
Infinite dimensional, 146
Information retrieval, 39, 218, 329, 362
Initial value problems, 311, 316
Inner product, 74, 244
complex inner product, 339
for C^n, 339
of functions, 244
of matrices, 244
of polynomials, 245
in R^n, 451
of vectors in R^n, 244
Inner product space, 244
complex, 339
norm for, 245, 249
Integrability, 558, 581
Integral
double, 558
improper, 647
in polar coordinates, 602–604
improper, 647–648
iterated, 553
line
scalar, see line integral, scalar, 657
vector, see line integral, vector, 659
properties of
linearity, 562
monotonicity, 562
triple, 580
improper, 647, 772
in cylindrical coordinates, 608–610
in spherical coordinates, 610–612
Interpolating polynomial, 238
Lagrange, 286
Invariant subspace, 309, 345
Inverse
computation of, 62–63
of an elementary matrix, 60
of a product, 52
Inverse matrix, 51, 463
Invertible matrix, 51
Involute, 417, 542
Involution, 57
Irreducible matrix, 388
Isobars, 526
Isomorphism
between row space and column space, 232
between vector spaces, 123
Isotherms, 526
Iterated integral, 553

J
Jacobi identity, 441, 480
Jacobi polynomials, 285
Jacobian, 598, 607
Jordan canonical form, 328
Judo, 408–409

K
Kepler, Johannes, 495
Kernel, 180
Kirchhoff’s laws, 19
Klein bottle, 739

L
Lagrange’s interpolating formula, 286
Laguerre polynomials, 285
Lamina, 622
Laplace’s equation, 701
Laplacian operator, 538
inversion formula for, 761
Latent semantic indexing, 220
Law of conservation of energy, 702
LDL’ factorization, 382
Lead variables, 13
Leading principal submatrix, 379
Least squares problem(s), 234–243, 259
Ceres orbit of Gauss, 235
fitting circles to data, 241
Least squares problem(s), solution of, 235
from Gram–Schmidt QR, 277
from normal equations, 237
Left inverse, 167
Left singular vectors, 354
Legendre polynomials, 284
Legendre, A. M., 234
Leibniz’s rule, 647
Length, 506
of a complex scalar, 339
in inner product spaces, 245
of a vector in C^n, 339
of a vector in R^n, 107, 117, 211
of a vector in R^n, 217
Length of a walk, 55
Leontief input–output models
closed model, 21–23, 390
open model, 386–388
Leslie matrix, 49
Leslie population model, 49
Line(s)
parametric equations of, see parametric equations, of a line, 415
skew, 448
Line integral
differential form, 663
numerical approximation of, 669–674
path-independent, 687, 688
scalar, 657
vector, 659
Line segment, 478
Linear combination, 34, 127
Linear differential equations
first order systems, 310–315
higher order systems, 315–319
Linear equation, 1

M
Machine epsilon, 360
Magnetic field
of a moving charge distribution, 765
of a moving point charge, 764
Magnetic monopoles, 766, 768
Manifold, 504
Mapping, linear, 457
Markov chain(s), 42, 152, 325–329, 390
Markov process, 42, 152, 325
MATLAB, 784–795
array operators, 792
built in functions, 789
entering matrices, 785
function files, 790
graphics, 792
help facility, 78, 794
M-files, 789
programming features, 789
relational and logical operators, 791
script files, 789
submatrices, 786
symbolic toolbox, 793
MATLAB path, 790
Matrices, 432–434, 453–460
addition of, 29
adjoint, 463
cofactor, 459
cofactor expansion, 459
determinant, 458
elementary row operations, 462
equality of, 29
Hilbert, 481
identity, 461

Linear mapping, see mapping, linear, 457
Linear operator, 175
Linear system(s), 1
equivalent, 58
homogeneous, 20
inconsistent, 2
matrix representation, 32
overdetermined, 14
underdetermined, 15
Linear transformation(s), 174–203
contraction, 190
definition, 174
dilation, 190
image, 180
inverse image, 183
kernel, 180
one-to-one, 183
onto, 183
range, 180
reflection, 190
from R^n to R^m, 178
standard matrix representation, 184
Linearly dependent, 135
Linearly independent, 135
in C^n, 141–143
in P^n: 140–141
Loggerhead sea turtle, 48, 80
Lorentz force, 676
Lower triangular, 63
LU factorization, 65
Matrices (continued)
inverse, 463
invertible, 463
matrix product, 455
properties, 456
minor, 459
multiplication of, 35
nilpotent, 481
nonsingular, 463
row equivalent, 61
scalar multiplication, 29, 455
properties, 455
similar, 200
transpose, 456
triangular, 462

Matrix
coefficient matrix, 7
column space of, 160
correlation, 222
covariance, 222–223
defective, 324
definition of, 7
determinant of, 93
diagonal, 64
diagonalizable, 321
diagonalizing, 321
elementary, 58
Fourier, 267
Hermitian, 341
identity, 50
inverse of, 51
invertible, 51
irreducible, 388
lower triangular, 63
negative definite, 373
negative semidefinite, 373
nonnegative, 386
nonsingular, 51
normal, 348
null space of, 126
orthogonal, 256
positive, 386
positive definite, 373
positive semidefinite, 373
powers of, 46
projection, 237, 261
rank of, 160
reducible, 388
row space of, 160
singular, 52
symmetric, 39
transpose of, 38
triangular, 63
unitary, 341
upper triangular, 63
Matrix algebra, 44–57
algebraic rules, 44
notational rules, 38
Matrix arithmetic, 27–44
Matrix exponential, 332
Matrix factorizations
Cholesky decomposition, 383
Gram–Schmidt QR, 275
LDU, 382
LU factorization, 65
Schur decomposition, 343
singular value decomposition, 351
Matrix generating functions, 786
Matrix multiplication, 35
definition, 35
Matrix norms
Frobenius, 247
Matrix notation, 27
Matrix representation theorem, 187
Matrix, adjoint of, 103
Maximum
local, 377
of a quadratic form, 374
Maxwell’s equations, 762–768
Mean value theorem, 571
for double integrals, 641, 776
for triple integrals, 747
Minimum
local, 377
of a quadratic form, 374
Minor, 91, 459
Mixtures, 312
Möbius strip, 730
Modified Gram–Schmidt process, 279
Moment, 619
first, 619, 775
of inertia, 626–628, 699, 776
second, 626–628
total, 619
Monopoles
magnetic, see magnetic monopoles, 766
Moving frame, 513, 519

\( n \)th root of unity, 271
Negative correlation, 222
Negative definite
matrix, 373
quadratic form, 373
Negative semidefinite
matrix, 373
quadratic form, 373
Nephroid, 483
Net force, 410
Networks
communication, 54
electrical, 19
Newton, Isaac, 496
Newtonian mechanics, 107
Nilpotent, 308
Nilpotent matrix, 481
Nonnegative matrices, 386–392
Nonnegative vector, 386
Nonorientable, 729
Nonsingular matrix, 51, 61
Norm
1-norm, 250
in \( \mathbb{C}^n \), 339
infinity, 250
from an inner product, 245, 249
of a vector, 249
Normal derivative, 701, 757, 770
Normal equations, 237
Normal matrices, 347–348
Normal plane, see plane, normal, 523
Normal spherical image, 544
Normal vector, 215
Normalization, 425
Normed linear space, 249
Null space, 126
dimension of, 162
Nullity, 163
Numerical integration, 286
for functions of one variable, 630–633
for functions of two variables, 633–642
Monte Carlo method, 641
Numerical rank, 360

O
Ohm’s law, 19
One-sided, see nonorientable, 729
Operation count
evaluation of determinant, 100
Opposite path, 665
Ordered basis, 150
Orientable, 729
Orientation, 668, 729
consistent, 740
induced, 740
Oriented, 668
Orthogonal, 423
Orthogonal complement, 227
Orthogonal matrices, 256–259
definition, 256
permutation matrices, 258
properties of, 258
Orthogonal polynomials, 281–288
Chebychev polynomials, 285
definition, 282
Hermite, 285
Jacobi polynomials, 285
Laguerre polynomials, 285
Legendre polynomials, 284
recursion relation, 283
roots of, 287
Orthogonal set(s), 253
Orthogonal subspaces, 226
Orthogonality
in \( n \)-space, 217
in an inner product space, 245
in \( \mathbb{R}^2 \) or \( \mathbb{R}^3 \), 213
Orthonormal basis, 255
Orthonormal set(s), 253–271
Osculating plane, see plane, osculating, 513, 522
Ostrogradsky’s theorem, see Green’s theorem, 684
Ostrogradsky, Mikhail, 684
Outer product, 74
Outer product expansion, 74
from singular value decomposition, 358, 362
Overdetermined, 14

P
Page rank, 329
PageRank algorithm, 329
Parallel axis theorem, 649
Parallellogram, 409
State vectors, 326
Stationary point, 372
Steady-state vector, 298
Stereographic projection, 774
Stochastic matrix, 152, 326
Stochastic process, 325
Stokes’s theorem, 679, 740–743
proof of, 750–753
Strake, 543
Strict triangular form, 5
Subspace(s), 123–132
definition, 124
Surface
of revolution, see surface of revolution, 775
oriented, 731
Surface area, 714–717
Surface area element, 735
Surface integrals, 719–738
scalar, 720, 735
vector, 724, 736
Surface of revolution, 775
Sylvester’s equation, 350
Symmetric matrix, 39

T
t-coordinate curve, 708
Tangent line, to a path, 493
Tangent plane
parametric equations for, 719
to a smooth parametrized surface, 712
Tangent spherical image, 544
Telegrapher’s equation, 772
Thermal conductivity, 770
Tofu, 582
Torque, 436, 545, 627
Torsion, 514–516, 519–521
Torus, 504, 709
Trace, 203, 253, 307
Tractrix, 542
Transition matrix, 153, 157
for a Markov process, 326
Translations, 191
Transpose, 456
of a matrix, 38
Trapezoidal rule
for functions of one variable, 630
for functions of two variables, 634
Triangle inequality, 249, 453
Triangular factorization, 64–65, see LU factorization
Triangular matrix, 63
Trigonometric polynomial, 263
Triple integral, 580
in cylindrical coordinates, 608–610
in spherical coordinates, 610–612
Trivial solution, 20
Tukey, J. W., 267
Two-sided, see orientable, 729