Math. 401, Sec. 500 Spring, 2005

Homework 10, due April 8

1. *[Schaum's, p. 46, Ex. 2.46 and 2.48)]* Use Parseval's identity along with Exercise 5 of Homework 8 and Exercise 6 of Homework 9 to show (in any convenient order)

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$.

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^6} = \frac{\pi^6}{945}$.

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n-1)^4} = \frac{\pi^4}{96}$.

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n-1)^6} = \frac{\pi^6}{960}$.

2. *[Schaum's, p. 46, Ex. 2.56 and 2.57)]*

   (a) A square plate of side L has one side maintained at temperature $f(x)$ and the others at zero. Find the steady-state temperature at any point of the plate (as a Fourier series of appropriate type).

   (b) Explain how to solve the problem if the four sides are maintained at temperatures $f_1(x)$, $g_1(y)$, $f_2(x)$, and $g_2(y)$. (Write out the answer in full for the case $f_2 = 0 = g_2$.)

3. *[Schaum's, p. 46, Ex. 2.58(a))]* An infinitely long plate of width L has its two parallel sides maintained at temperature 0 and its other side at constant temperature T. Find the steady-state temperature.

4. *[Schaum's, p. 46, Ex. 2.63)]* Solve the boundary value problem

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - \alpha^2 u, \qquad u(0,t) = u_1\,, \quad u(L,t) = u_2\,, \quad u(x,0) = 0,$$

   where $0 < x < L$, $0 < t$, and α, L, u_1, and u_2 are constants.

   Instructions for Exercises 5 and 6: J. B. Fourier was ridiculed by some of the mathematicians of his day when he first announced his discovery that an arbitrary function on the interval $0 < x < \pi$, such as $f(x) = x^2$, can be expanded in a series of sine functions. Some of the criticisms were like the two statements which follow. In each case explain in a short essay how the mathematicians were confused (and Fourier was right).

5. "x^2 is an even function; but any fool can see that a sum of sines will always be an odd function."

6. "x^2 is not zero at the right endpoint (π); but any fool can see that a sum of the functions $\sin nx$ will always vanish at $x = \pi$. The same criticism applies if we consider the limits of functions as $x \to \pi$ instead of the values of the functions when $x = \pi$."