Math. 401, Sec. 500 Spring, 2005

Homework 4, due February 16

1. *[Logan, p. 49, Ex. 1.19]* Apply the Poincaré–Lindstedt method to the scaled pendulum problem (stated in the last exercise of Homework 3). Find a two-term perturbation solution.

2. By the Poincaré (distorted-time) method, find a good one-term approximation to the solutions of

$$\frac{d^2y}{dt^2} + [\omega^2 - V(t)]y = 0 \tag{1}$$

when $\omega \to +\infty$ (i.e., $\epsilon = \omega^{-1}$ is the small parameter), and

(a) $V(t) = \dfrac{1}{1+t^2}$.

(b) V is an arbitrary continuous function.

In both cases, assume that $\omega^2 - V(t)$ is positive in the interval of interest. (See Problem 4 below for more about that.)

3. (a) Apply the Poincaré method to

$$\frac{d^2y}{dt^2} + \epsilon\left(\frac{dy}{dt}\right)^3 + y = 0, \qquad y(0) = 1, \quad \frac{dy}{dt}(0) = 0.$$

Carry the calculations far enough to demonstrate that that method *doesn't work* on this problem.

(b) Treat the problem by the method of two time scales. *Hint for solving the two coupled nonlinear equations:* Try setting one function identically equal to 0.

(c) Why did method (a) fail and method (b) work? [What makes this equation different from, say, $y'' + y + \epsilon y^3 = 0$?]

4. In our study of the equation $\dfrac{d^2y}{dt^2} + [\omega^2 - V(t)]y = 0$ we assumed that $\omega^2 - V(t)$ is positive. Give a critical discussion of the following claim:

This condition will always be satisfied if ω is sufficiently large.

Is it true, false, or a half-truth? *Hint:* Remember the distinction between pointwise and uniform limits. (It is understood that ω is real.)