Math. 401, Sec. 501 Spring, 2006

Homework 2, due February 1

In Exercises 1–4 find the leading behavior of all roots. For real roots, continue the calculation up through the first positive power of ϵ.

1. *[Logan, p. 59, 2.1(a)]* $\epsilon x^4 + \epsilon x^3 - x^2 + 2x - 1 = 0$ (Finish the solution started in the notes, pp. 14–17.)
2. *[Simmonds, p. 37, 2.6(a)]* $1 + \epsilon^{-1} z + \epsilon^{-1} z^2 + z^3 = 0$
3. *[Simmonds, p. 37, 2.6(b)]* $1 - 2z + z^2 + \epsilon z^5 = 0$
4. *[Bush, p. 17, (v)]* $(x^2 - 1)(x^2 - 4)(x - 3) + \epsilon = 0$
5. A bridge spans a horizontal distance of 5000 feet. The shape of the bridge is a circular arc, and its total length is exactly one foot greater than the straight-line distance it spans. What is the height of the midpoint of the bridge? (In geometrical language: What is the distance from an arc to its chord if the arc length is 5001 units and the chord length is 5000 units?) Try to get an answer accurate to 3 significant figures; you should find that easier to do by perturbation theory than by the quadratic formula. The point is to do this *without* a calculator. Therefore, do *not* subtract two nearly equal quantities. Also, do *not* evaluate trig or inverse trig functions numerically; instead, use
$$\sin\theta \approx \theta - \frac{\theta^3}{3!}$$
to get a solvable quadratic equation for the angle. Then use perturbation theory to solve the quadratic equation for the height. (For help with the geometry, see F. S. Acton, *Numerical Methods that Work*, pp. 3–4 and 67–69. Acton also gives a numerical (iterative) solution method, which is ultimately equivalent to the perturbative method.)
6. *[Logan, p. 49, Ex. 1.4]* Find the first two terms of the perturbation series solution to the initial value problem
$$\frac{dy}{dt} = 1 + (1+\epsilon)y^2, \qquad y(0) = 1$$
(Finish the solution started in the notes, pp. 20–21.) Find the exact solution and compare the approximation. Is it uniform?
7. Find the first two terms of the perturbation series solution to the initial value problem
$$\frac{d^2y}{dt^2} + \epsilon\frac{dy}{dt} + 9y = 0, \qquad y(0) = 1, \quad y'(0) = 0.$$
Find the exact solution and compare the approximation. Is it uniform?