Math. 401, Sec. 501 Spring, 2009

Homework 1, due January 28

In Exercises 1 and 2, find approximate solutions of the form $x \approx x_0 + \epsilon x_1$ (ϵ small).

1. $x^3 + \epsilon x^2 + 1 = 0$

2. $x^5 + \epsilon x - 32 = 0$

3. Consider $x^2 + 2\epsilon x - 1 = 0$.

   (a) Find approximate solutions of the forms $x \approx x_0 + \epsilon x_1$ and $x \approx x_0 + \epsilon x_1 + \epsilon^2 x_2$.

   (b) Check the consistency of your answers to (a) with the Taylor expansion of the exact solution.

   (c) Compare the first-order, second-order, and exact solutions numerically, for $\epsilon = 10$, 1, 0.1, and 0.01.

4. Find the second-order solutions ($x \approx x_0 + \epsilon x_1 + \epsilon^2 x_2$) to $x^4 + \epsilon x - 1 = 0$.