Math. 401, Sec. 501 Spring, 2009

Homework 6, due March 6

1. *[Bush, p. 156, (i) and (ii)]* Find the (lowest-order) composite expansion for the solution of

   (a) $\epsilon y'' + y' + \dfrac{y}{x+1} = 2.$

   (b) $\epsilon y'' - y' + \dfrac{y}{x+1} = 2.$

   In both cases [(a) and (b)] consider

   $$0 < \epsilon \ll 1, \qquad 0 < x < 1, \qquad y(0) = 0, \quad y(1) = 3.$$

2. *[Bush, p. 156, (iii)–(v)]* Find the composite expansion for the solution of

   (a) $\epsilon y'' + xy' + y = 0,$

   $$0 < \epsilon \ll 1, \qquad 2 < x < 4, \qquad y(2) = 0, \quad y(4) = 1.$$

   (b) $\epsilon y'' - xy' + y = 0,$

   $$0 < \epsilon \ll 1, \qquad 2 < x < 4, \qquad y(2) = 0, \quad y(4) = 1.$$

   (c) $\epsilon y'' + xy' + y = 0,$

   $$0 < \epsilon \ll 1, \qquad -4 < x < -2, \qquad y(-4) = 1, \quad y(-2) = 0.$$

   Do not work out (c) from the beginning; knowing the answers to (a) and (b), you can solve (c) in two or three lines by a change of variable.

3. *[Bush, p. 172, (i)]* $\epsilon y'' - y' + \dfrac{1}{y} = 0,$

   $$0 < \epsilon \ll 1, \qquad 0 < x < 1, \qquad y(0) = 2, \quad y(1) = 1.$$

4. *[Bush, p. 172, (ii)]* $\epsilon y'' + y' + e^y = 0,$

   $$0 < \epsilon \ll 1, \qquad 0 < x < 1, \qquad y(0) = 1, \quad y(1) = -\ln 2.$$

5. *[Logan, p. 69, Ex. 3.2(a)]* Find the lowest-order composite expansion for positive ϵ. Compare the result with the exact solution, either numerically or in terms of power series.

   $$\epsilon y'' + 2y' + y = 0, \qquad y(0) = 0, \quad y(1) = 1.$$

6. *[Logan, p. 69, Ex. 3.4]* Find the lowest-order composite expansion for positive ϵ. Compare the result with the exact solution, either numerically or in terms of power series.

$$\epsilon y'' - y' = 2t, \qquad y(0) = y(1) = 1.$$

7. *[Logan, p. 69, Ex. 3.9(a)]* $\epsilon y'' + (t^2 + 1)y' - t^3 y = 0,$

$$0 < \epsilon \ll 1, \qquad y(0) = y(1) = 1.$$

8. *[Logan, p. 69, Ex. 3.9(b)]* $\epsilon y'' + (\cosh t)y' - y = 0,$

$$0 < \epsilon \ll 1, \qquad y(0) = y(1) = 1.$$

Hint: You may find some help with the integral in a handbook under "Gudermannian function" or "Lobachevsky's angle of parallelism".