Math. 401, Sec. 501 Spring, 2009

Homework 7, due March 11

1. Classify these equations as linear homogeneous, linear nonhomogeneous, or nonlinear. (Here $u_t \equiv \partial u/\partial t$, etc.)

   (a) $u_{tt} - u_{xx} = \cos(x - t)$

   (b) $u_t + u^2 u_x = 0$

   (c) $u_t + 3t^2 u = 0$

   (d) $u_{tt} - u_{xx} = -m^2 u$

2. Why do we not make a distinction between "homogeneous" and "nonhomogeneous" for *nonlinear* equations? *Hint:* Try to classify the equation $y'' + (\cos y)^2 = 1$.

3. *[Schaum's, p. 46, Ex. 2.52]* Find the steady-state temperature in a bar whose ends are located at $x = 0$ and $x = 10$, if these ends are kept at 150°C and 100°C respectively.

4. Suppose that the boundary conditions ("BC:") on p. 65 of the notes (and corresponding lecture on the heat equation with fixed end temperatures) are replaced by

$$\frac{\partial u}{\partial x}(t, 0) = F_1\,, \qquad \frac{\partial u}{\partial x}(t, 1) = F_2\,.$$

   (That is, the *heat flux* through each end of the bars is held constant.) What happens when you attempt to find a steady-state solution as on p. 67? Distinguish between the two cases $F_1 = F_2$ and $F_1 \neq F_2$. Can you give a physical explanation for your results?