Yet another big example (with infinite domain)

$$
\begin{gathered}
\frac{\partial^{2} u}{\partial x^{2}}+\frac{\partial^{2} u}{\partial y^{2}}=0 \quad(0<x<\infty, \quad 0<y<\pi) \\
u(0, y)=g(y) \quad(0<y<\pi) \\
u(x, 0)=0 \quad(0<x<\infty) \\
u(x, \pi)=f(x) \quad(0<x<\infty)
\end{gathered}
$$

