Team Delta

Proof of Proposition 3.6

<u>Proposition 3.6</u>: Given A*B*C. Then B is in the only point common to rays BA and BC, and ray AB= ray AC.

Proof: To prove the first statement of proposition 3.6 the following proof was developed.

There exist 3 collinear points A, B, & C such that A*B*C and there exists a point P on the line AC such that P ≠ B, and P is on rays BA & BC.

For P on ray BA either P=B, P=A, B*A*P, or B*P*A.

By (1) P≠B and if P=A, B*A*P, or B*P*A then P is not on ray BC

By (3) P cannot lie on both rays BA & BC therefore P must equal B to be on both rays which refutes (1) (RAA).

Furthermore, to prove the second statement in Proposition 3.6, the following proof was developed.

Proof:

1. There exists three collinear points A, B, and C such that A*B*C, the following proof will show that AB is a subset of AC and that AC is a subset of AB.
2. For P included in ray AB, either (a) P=A, (b) P=B, (c) A*P*B, (d) A*B*P.
3. For case 2a P is on ray AC by the definition of a ray.
4. For case 2b P is on ray AC by A*B*C and the definition of a ray.
5. For case 2c P is on the ray AC by A*B*C and Proposition 3.3 (either A*P*C or A*C*P).
6. For case 2d P is on the ray AC by A*B*C and Proposition 3.3.
7. By (3), (4), (5), (6) set AB is a subset of AC proving the first assumption in (1).
8. For P included in ray AC, either (a) P=A,(b) P=C, (c) A*P*C, (d) A*C*P.
9. For case 2a P is on ray AB by the definition of a ray.
10. For case 2b P is on ray AB by A*B*C and the definition of a ray.
11. For case 2c P is on the ray AB by A*B*C and Proposition 3.3 (either A*P*B or A*B*P).
12. For case 2d P is on the ray AB by A*B*C and Proposition 3.3.
13. By (9), (10), (11), (12) set AC is a subset of AB proving the second assumption in (1).

14. By (7) and (13) ray AB is congruent to ray AC.