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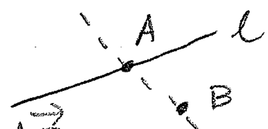
Math 467
Team: Gamma

Team Gamma

Proposition 3.7 = ex.10 (Requires Ex. 9)

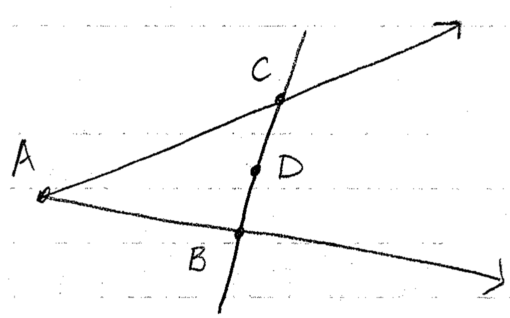
exercise 9 states: Given a line l, a point A on l, and a point B not on l. Then every point of the ray $\overrightarrow{AB}$ (except A) is on the same side of l as B. Here the ray $\overrightarrow{AB}$ is the set of all points X on the line $\overleftrightarrow{AB}$ such that either X is on AB or A * B * X.



Proof: (RAA argument)
There exist a point C on the ray $\overrightarrow{AB}$, C ≠ A and C is on the opposide side of l. ∴
C ≠ B since B is not on the opposite side of itself.
The segment BC intersects l at A. (By the Def. of Opposite Sides) Now C is on $\overrightarrow{AB}$ iff
C = A or C = B or A * C * B or A * B * C
A = C + B = C are already excluded as stated above.
So by Axiom B- , C * A * B cannot occur.
This contradicts that C is on the opposite of l from B. So CB does not intercept l, and B and C are on the same side of l.

exercise 10 states: Prove Proposition 3.7 which says → Given an angle ∢CAB and point D lying on line $\overleftrightarrow{BC}$. Then D is in the interior of ∢CAB if and only if B * D * C

example figure from book



Proof:

By hypothesis, D lies on $\overleftrightarrow{BC}$ and B is on line $\overleftrightarrow{AB}$ and C is on line $\overleftrightarrow{AC}$.

By proposition 3.1, ray $\overrightarrow{BC} \cup \overrightarrow{CB} = \{\overleftrightarrow{BC}\}$

Therefore D lies on $\overrightarrow{BC}$ or lies on $\overrightarrow{CB}$

From exercise 9 proof

As previously proved, if D lies on $\overrightarrow{BC}$ and B is on line $\overleftrightarrow{AB}$ then D lies on the same side of $\overleftrightarrow{AB}$ as C.

As previously proved, if D lies on $\overrightarrow{CB}$ and C is on line $\overleftrightarrow{AC}$ then D lies on the same side of $\overleftrightarrow{AC}$ as B.

By Axiom B-3, one and only one of pts. B, C, D are between the other two.

Assume $B * D * C$, therefore D lies on $\overrightarrow{BC}$ and $\overrightarrow{CB}$ by def. Therefore, as previously stated, D is on the same side of line $\overleftrightarrow{AC}$ as B <u>and</u> the same side of line $\overleftrightarrow{AB}$ as C. Therefore, by def. of the interior of an angle, D lies on the interior of $\measuredangle CAB$, for $B * D * C$.

Now if $D = B$, then D would lie on $\overrightarrow{AB}$ and $\therefore$ would not lie on either side of line $\overleftrightarrow{AB}$ and $\therefore$ it would not lie in the interior of $\measuredangle CAB$.

Similarly, if $D = C$, then D would lie on $\overrightarrow{AC}$ and $\therefore$ would not lie on either side of line $\overleftrightarrow{AC}$ and $\therefore$ it would not lie in the interior of $\measuredangle CAB$.

Now if $D * B * C$, then by def. of opposite sides, line $\overleftrightarrow{AB}$ would intersect line $\overleftrightarrow{DC}$ $\therefore$ D and C would be on opposite sides of line $\overleftrightarrow{AB}$ so $\therefore$ D would not lie in the interior of $\measuredangle CAB$.

Similarly, ~~$\not$~~, $B * C * D$, then by def. of opposite sides, line $\overleftrightarrow{AC}$ would intersect line $\overleftrightarrow{BD}$ $\therefore$ B and D would be on opposite sides of line $\overleftrightarrow{AC}$ so $\therefore$ D would not lie in the interior of $\angle CAB$.

So $\therefore$ since the cases for $D = C$, $D = B$, $B * C * D$, and $D * B * C$ do not work out and the case where $B * D * C$ is the only case that stands true we can conclude that, D lies on the interior of $\angle CAB$ if and only if $B * D * C$. ∎