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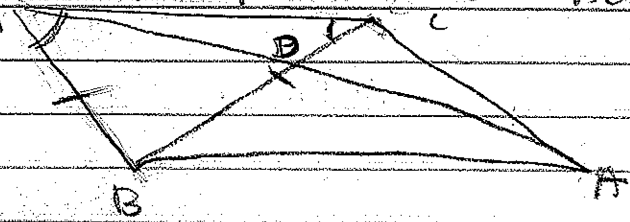
Proposition 4.6

Given $\triangle ABC$ and $\triangle A'B'C'$, if we have $AB \cong A'B'$ and $BC \cong B'C'$ then $\measuredangle B < \measuredangle B'$ iff $AC < A'C'$

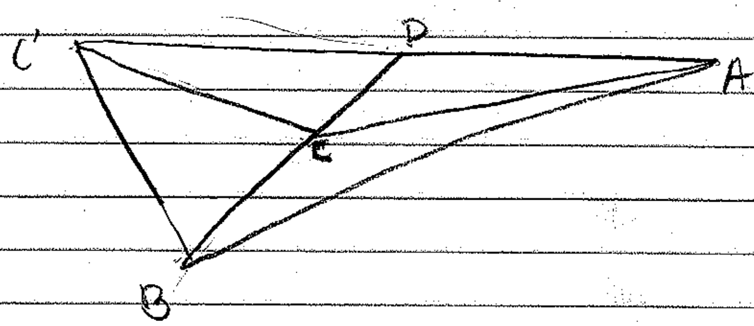
$\Rightarrow$ Given $\measuredangle B < \measuredangle B'$ show $AC < A'C'$

Pf: By axiom C-1 we reduce to the case where $A = A'$, $B = B'$, $BC \cong BC'$ and C is interior to $\measuredangle ABC'$. Therefore we want to show that $AC < AC'$. By the crossbar Thm BC intersects AC' ~~at~~ pt D



Case 1: $C = D$. Then $A * C * C'$ and therefore $AC < AC'$

Case 2: Suppose $C \neq D$. By proposition 4.5 we want to show that $\measuredangle AC'C < \measuredangle ACC'$. By Isosceles triangles $\measuredangle BC'C \cong \measuredangle BCC'$. In the case where $B * D * C$ then $\measuredangle AC'C < \measuredangle BC'C$ and $\measuredangle BCC' < \measuredangle ACC'$. By transitivity $\measuredangle AC'C < \measuredangle ACC'$. Therefore $AC < AC'$ by proposition 4.5. In the case where $B * C * D$ we must apply the Exterior Angle Theorem twice:



$\measuredangle ACC' > \measuredangle DCC' > \measuredangle BCC' \cong \measuredangle BCC' > \measuredangle CC'D = \measuredangle AC'C$

By transitivity $\measuredangle ACC' > \measuredangle AC'C$ and by prop 4.5 $AC < AC'$

$\Leftarrow$ Given $AC' > AC$. Show $\sphericalangle ABC < \sphericalangle ABC'$

(RAA hypothesis) Suppose $\sphericalangle ABC > \sphericalangle ABC'$.

We have already shown that $\sphericalangle B > \sphericalangle B' \Rightarrow AC > AC'$

By trichotomy only one of $AC > AC'$, $AC \cong AC'$, or $AC < AC'$ holds. therefore the RAA hypothesis violates the $\Rightarrow$ statement. therefore if $AC' > AC$ then $\sphericalangle ABC < \sphericalangle ABC'$