Proposition 4.7

Team Delta

Proposition 4.7: Hilbert's Euclidian parallel postulate if a line intersects one of two parallel lines, then it also intersects the other.

=>

Suppose 2 lines *l* and *m* are parallel, and a third line *n* intersects line *m*.

By proposition 2.5 there exists a unique point P incident with *n* and *m*.

The Hilbert axiom of parallelism explains that since *m* is incident with P, and *l* is parallel to *m*, then *n* is not parallel to *l* unless it is equal to *m*.

Therefore by definition of parallel *n* intersects *m* and *l*.

<=

Suppose there is a line *l* and a point P, not on *l*, such that *m* is incident with P, *n* is incident with P, and *l* is parallel to *m*.

Since *n* intersects *m* and because *m* is parallel to *l*, *n* must also intersect *l*. (By statement in proposition 4.7)

Therefore lines *l* and *n* are not parallel and line *m* is the unique line through P that is parallel to *l*, proving the Hilbert Euclidian parallel postulate.