Lindsay Mitchell

Allie Harrell

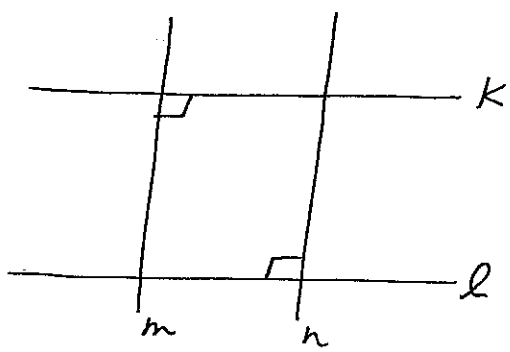
Math 467- Weeks 8 and 9

Discussion: Proposition 4.10

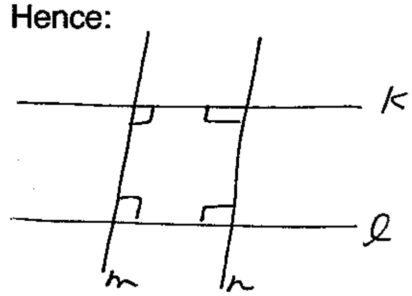
Proposition 4.10- Hilbert's Euclidean parallel postulate $\Leftrightarrow$ if $k \parallel l$, $m \perp k$, and $n \perp l$, then either $m = n$ or $m \parallel n$.

Proof:

a)

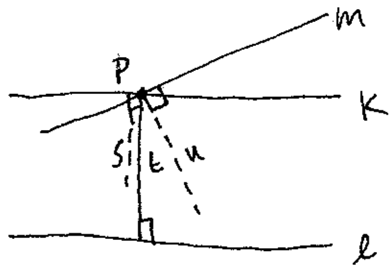


Suppose $k \parallel l$, $m \perp k$, and $n \perp l$. Since $m \perp k$, it follows then m intersects k and also l by Proposition 4.7. Since m is a transversal and $m \perp k$, then it is true that $m \perp l$ by the hypothesis and Proposition 4.9. Similarly, $n \perp l$ and $n \perp k$ by Proposition 4.7 and Proposition 4.9. Hence:



It is either true that $m = n$ or $m \neq n$. If we take $m \neq n$, it follows that $m \parallel n$ since $m \perp k$ and $n \perp k$ by Corollary 1 to the Alternate Interior Angle Theorem. Thus, it is understood that $m \neq n$ by the same Corollary.

⇐ Proof:



We need to show that ~~a~~ $m=k$ by showing that $s=u$. Let t be the $\perp$ from P to l. Let s be the $\perp$ to k at P. Let u be the $\perp$ to m at P. From here, we will show that $s=t$ and $u=t$. First, we can conclude that either $s=t$ or $s \parallel t$. In addition, either $u=t$ or $u \parallel t$. We know that s and t cannot be parallel, and u and t cannot be parallel, since all the lines go through point P. Thus $s=u$, and therefore $m=k$ by Congruence Axiom 4 and Euclid's 4th Postulate.