Adapted from Team Eta (Kirk Nelson, Derek Tomlin) and Team Gamma (Yaneira Gonzalez Vergara, Eric Wolff)

Proposition 4.8

Hilbert's Euclidean parallel postulate is equivalent to the converse of the alternate interior angle theorem.

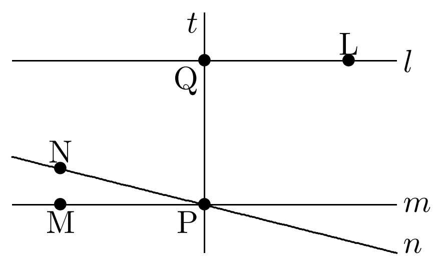
In more detail:

For every line l and every point P not lying on l there is at most one line m through P such that m is parallel to l.

$$\iff$$

If twe lines are parallel and are cut by a transversal t, then they have a pair of congruent alternate interior angles with respect to t.

$\Rightarrow$: Assume first the parallel postulate and suppose we have the situation in the figure below. Lines l and m are parallel, with transversal t through P. Construct n through P so that $\angle$NPQ $\cong$ $\angle$LPQ. By the ordinary (direct) AIA theorem, n is parallel to l. But HE says that the parallel to l through P is unique, so $m = n$. Then the alternate interior angles of l and m about t are congruent, QED.



$\Leftarrow$: Now assume the AIA converse. Let m and n be parallel to l through P, and let t be a transversal through P intersecting l at Q. By the AIA converse, $\angle$PQL $\cong$ $\angle$QPN and $\angle$PQL $\cong$ $\angle$QPM, so $\angle$QPN $\cong$ $\angle$QPM by transitivity. Axiom C-4 says that for a given angle, the ray from point P on t is unique, so $n = m$. Thus HE holds.