Math. 467 (Fulling) 13 October 2010

Midterm Test – Solutions

Name: _____

1. *(Multiple choice – each 5 pts.) (Circle the correct capital letter.)*

(a) Euclid lived before

(A) Thales
(B) Plato
(C) Proclus
(D) Pythagoras

C (the *last* of the famous ancient Greek geometers, who wrote commentary on Euclid)

(b) The *definition* of "opposite rays" includes all of the following conditions except which?

(A) They are part of the same line.
(B) Their union contains all the points on the line.
(C) They are distinct.
(D) They emanate from the same point.

B (We had to prove this one as a theorem, Prop. 3.4.)

(c) Euclid was unable to provide a straightedge-and-compass construction of

(A) a trisected angle.
(B) a perpendicular to a line at a given point on the line.
(C) a regular pentagon inscribed in a circle.
(D) a tangent to a circle from a point outside.
(E) a square on a given segment as side.

A (a famous *unsolvable* problem)

(d) The Greek approach to geometry was revolutionary because

(A) they discovered the Pythagorean theorem and divided a circle into 360 degrees.
(B) they insisted upon proofs.
(C) they were not interested primarily in practical applications.
(D) they focused upon idealizations such as infinitely thin and infinitely straight lines.
(E) [all of these except (A)]

E (The Babylonians had already done A (p. 2).)

(e) Which of the following is *not* provable from the Hilbert I and B axioms?

(A) If $\overrightarrow{AD}$ is between $\overrightarrow{AC}$ and $\overrightarrow{AB}$, then $\overrightarrow{AD}$ intersects segment BC.

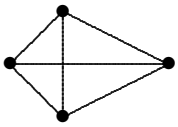
(B) If D lies on line $\overleftrightarrow{BC}$, then D is in the interior of $\angle$CAB if B * D * C.

(C) If D lies on line $\overleftrightarrow{BC}$, then B * D * C if D is in the interior of $\angle$CAB.

(D) Every point D in the interior of $\angle$CAB lies on a segment joining a point E on $\overrightarrow{AB}$ to a point F on $\overrightarrow{AC}$.

(E) If D is in the interior of $\angle$CAB, then so is every other point on $\overrightarrow{AD}$ except A.

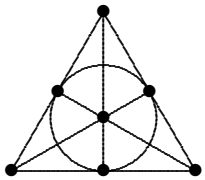
D (Recall the "warning" on p. 115. The others are propositions nearby it.)

(f)  This diagram is used in the proof of

(A) Pasch's theorem
(B) crossbar theorem
(C) SAS
(D) ASA
(E) SSS

E

BONUS QUESTIONS FOR R STUDENTS ONLY

(g) Which statement is false?  This geometry with 7 points and 7 lines

(A) is called the *Fano plane*.
(B) is the smallest projective plane.
(C) satisfies the Hilbert incidence axioms.
(D) has the Euclidean parallel property.

D

(h) Euclid's geometry is based on five fundamental, undefined terms, but the importance of one of them was not fully realized until the work of Pasch and Hilbert over 2000 years later. Which one?

(A) point
(B) line
(C) lie on
(D) between
(E) congruent

D

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2. *(13 pts.)* Give (describe) a finite model of the three incidence axioms that satisfies the Euclidean parallelism property. (There is a standard one.) Justify your answer briefly.

The model has 4 points, and the lines are all the 2-point subsets. See Example 3 and Fig. 2.5, pp. 74–75.

3. *(10 pts.)* Simplify $\neg\exists x\,\forall m\,[x \geq 0 \wedge S(m,x) \wedge \forall n\, Q(m,n,x)]$.
(Push the "$\neg$" in as far as you can!) (In Greenberg's notation, $\neg$ is $\sim$, and $\wedge$ is &.)

$$\forall x \exists m [x < 0 \vee \neg S(m,x) \vee \exists n \big(\neg Q(m,n,x)\big)]$$

4. *(21 pts.)* State the three congruence axioms that concern only segments (not angles).

See pp. 119–120.

5. *(Essay – 26 pts.)* Recall *Pasch's theorem*: If A, B, C are distinct noncollinear points and l is any line intersecting AB in a point between A and B, then l also intersects either AC or BC; if C does not lie on L, then l does not intersect both AC and BC.

   (Use either back side or separate paper.) Write an essay about this theorem, covering (at least) these points:

   - a quick summary of the meaning or significance of the theorem;
   - the proof;
   - an explanation of why the intersection point is required to be in the interior of AB. (What possible things could happen if the line hits an endpoint instead?)