Math. 467 (Fulling) 17 October 2014

Midterm Test – Solutions

Name: ______________________________

1. *(Multiple choice – each 5 pts.) (Circle the correct capital letter.)*

   (a) $p \Rightarrow q$ is equivalent to
   - (A) $p \vee \neg q$
   - (B) $\neg(q \wedge \neg p)$
   - (C) $q \vee \neg p$
   - (D) $\neg p \Rightarrow \neg q$
   - (E) none of these.

(C)

   (b) Which of the following is *not* provable from the Hilbert I and B axioms?
   - (A) If $\overrightarrow{AD}$ is between $\overrightarrow{AC}$ and $\overrightarrow{AB}$, then $\overrightarrow{AD}$ intersects segment BC.
   - (B) If D lies on line $\overleftrightarrow{BC}$, then D is in the interior of ∠CAB if B * D * C.
   - (C) Every point D in the interior of ∠CAB lies on a segment joining a point E on $\overrightarrow{AB}$ to a point F on $\overrightarrow{AC}$.
   - (D) If D is in the interior of ∠CAB, then so is every other point on $\overrightarrow{AD}$ except A.
   - (E) If D lies on line $\overleftrightarrow{BC}$, then B * D * C if D is in the interior of ∠CAB.

(C) (the famous “Warning”)

2. *(15 pts.)* List the three congruence axioms that involve angles. (These are the last three.)

[See pp. 120–121.]

3. *(12 pts.)* State the *crossbar theorem* and draw a sketch to illustrate it.

[See p. 116]

4. *(18 pts.)* Present a model of the three incidence axioms, containing only finitely many points, that satisfies the *hyperbolic* parallelism property. How much freedom do you have in choosing the number of points?

Take a set of *5 or more* points and define the lines to be the subsets containing exactly two points. Then the required 4 axioms are satisfied. See pp. 74–75 and Figure 2.6 for details.

5. *(7 pts.)* What is the definition of “opposite ray”?

Greenberg gives two definitions, which are equivalent. (1) (p. 18) Two rays are opposite if they are distinct, emanate from the same point, and are part of the same line. (2) (p. 109) If C * A * B, then $\overrightarrow{AC}$ and $\overrightarrow{AB}$ are opposite.

6. *(18 pts.)* State and prove **ONE** of these: (50% extra credit for the other one)
   - (A) Pasch’s theorem
   - (B) The SSS triangle congruence theorem

7. *(Essay – 20 pts.)* Proposition 2.3 is, "For every line, there is at least one point not lying on it." One of your teammates has proposed the following proof:

   > According to Axiom I-3, there are three points (call them A, B, and C) such that no line is incident with all of them. Let l be the line $\overleftrightarrow{\mathrm{AB}}$. Then C does not lie on l, QED.

   Explain what is wrong with this proof. Then give a correct proof.