Math. 467 (Fulling) 14 October 2016

Midterm Test – Solutions

Name: ______________________________

1. *(Multiple choice – each 5 pts.) (Circle the correct capital letter.)*

   (a) The smallest geometry that has the Euclidean parallelism property has

   (A) 2 points.
   (B) 3 points.
   (C) 4 points.
   (D) 5 points.
   (E) 6 points.

C

   (b) Moritz Pasch is known best for stressing the importance of the concept of

   (A) incidence.
   (B) betweenness.
   (C) congruence.
   (D) continuity.
   (E) parallelism.

B

   (c) The side-angle-side congruency criterion needs to be an axiom because

   (A) Euclid chose it as his most fundamental assumption.
   (B) without it there is no hope of proving the parallel postulate.
   (C) no other axiom relates segments to angles.
   (D) Euclid's proof of it is fallacious because he didn't understand "betweenness".
   (E) angle subtraction must be a theorem, but segment subtraction is part of one of the axioms.

C

   (d) In connection with models of axioms of geometry, **duality** refers to

   (A) the two-dimensional nature of the Euclidean plane.
   (B) interchange of points and lines.
   (C) interchange of segments and angles.
   (D) interchange of resonances and Regge poles.
   (E) interchange of the two sides of a line.

B (But D is a real Thing in string theory.)

(e) If $\angle$CBG $\cong$ $\angle$FEH and $\angle$GBA $\cong$ $\angle$HED, then $\angle$ABC $\cong$ $\angle$DEF, provided that

(A) $\angle$CBG < $\angle$GBA.
(B) none of the angles is obtuse.
(C) the Euclidean parallel postulate is true.
(D) $\overrightarrow{\text{BA}}$ and $\overrightarrow{\text{BC}}$ are not opposite rays.
(E) $\overrightarrow{\text{BG}}$ is between $\overrightarrow{\text{BA}}$ and $\overrightarrow{\text{BC}}$, and $\overrightarrow{\text{EH}}$ is between $\overrightarrow{\text{ED}}$ and $\overrightarrow{\text{EF}}$.

E

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2. *(10 pts.)* Simplify $\neg\forall x\,\exists m\,\big[x < 0 \vee S(m,x) \;\Rightarrow\; \exists n\,\big(T(m,n) \wedge n > x\big)\big]$.
(Push the "$\neg$" in as far as you can!) (In Greenberg's notation, $\neg$ is $\sim$, and $\wedge$ is &.)

$$\exists x\,\forall m \neg\,\big[x < 0 \vee S(m,x) \;\Rightarrow\; \exists n\,\big(T(m,n) \wedge n > x\big)\big]\,.$$

$$\exists x\,\forall m\,\Big[\big(x < 0 \vee S(m,x)\big) \wedge \neg\big(\exists n\,\big(T(m,n) \wedge n > x\big)\big)\Big]$$

$$\exists x\,\forall m\,\Big[\big(x < 0 \vee S(m,x)\big) \wedge \forall n\,\neg\big(T(m,n) \wedge n > x\big)\Big]$$

$$\exists x\,\forall m\,\Big[\big(x < 0 \vee S(m,x)\big) \wedge \forall n\,\big(\neg T(m,n) \vee n \le x\big)\Big]$$

3. *(25 pts.)* State the four **betweenness** axioms.

[See pp. 108–111.]

4. *(20 pts.)* State and prove either **Pasch's theorem** or the **crossbar theorem**. (Extra credit for doing both is limited to 10 points.)

[See pp. 114 and 116.]

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5. *(Essay – 20 pts.) IMPROVEMENTS IN THE REWRITE GET ONLY HALF CREDIT. E.G., IF YOUR INITIAL SCORE IS 16, YOUR FINAL SCORE WON'T EXCEED 18.*

Define **equivalence relation** and **partition**. Then state the theorem that says how these two things are related. (A full proof of the theorem is not required.)