Math. 467 (Fulling) 2 October 2020

Midterm Test – Solutions

Upload your answers, in order, as a single document.

1. *(Multiple choice – each 5 pts.) (Indicate the correct capital letter.)*

   (a) The Greek approach to geometry was revolutionary because

   (A) they insisted upon proofs.
   (B) they were not interested primarily in practical applications.
   (C) they focused upon idealizations such as infinitely thin and infinitely straight lines.
   (D) they discovered the Pythagorean theorem and divided a circle into 360 degrees.
   (E) [all of these except (D)]

E (D was known to the Babylonians.)

   (b) Which of the following is *not* provable from the Hilbert I and B axioms?

   (A) Every point D in the interior of $\angle$ CAB lies on a segment joining a point E on $\overrightarrow{\mathrm{AB}}$ to a point F on $\overrightarrow{\mathrm{AC}}$.
   (B) If $\overrightarrow{\mathrm{AD}}$ is between $\overrightarrow{\mathrm{AC}}$ and $\overrightarrow{\mathrm{AB}}$, then $\overrightarrow{\mathrm{AD}}$ intersects segment BC.
   (C) If D lies on line $\overleftrightarrow{\mathrm{BC}}$, then D is in the interior of $\angle$ CAB if B $*$ D $*$ C.
   (D) If D lies on line $\overleftrightarrow{\mathrm{BC}}$, then B $*$ D $*$ C if D is in the interior of $\angle$ CAB.
   (E) If A is on line l and B is not on l, then every point of $\overrightarrow{\mathrm{AB}}$ except A is on the same side of l as B.

A (“Warning”, p. 115.)

   (c) The importance of the concept of “betweenness” was established by

   (A) Euclid.
   (B) Playfair.
   (C) Pappus.
   (D) Pasch.
   (E) Proclus.

D

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2. *(18 pts.)* State the three Hilbert **congruence** axioms that involve angles. (These are the last three.)

[See pp. 120–121.]

3. *(12 pts.)* Simplify $\neg\exists x\,\forall y\ \big[\big(x \ge 0 \wedge x \le y\big) \ \Rightarrow\ \forall n\,\big(T(y,n) \vee \neg S(x,n)\big)\big]$.
   (Push the “ $\neg$ ” in as far as you can!) (In Greenberg’s notation, $\neg$ is $\sim$, and $\wedge$ is &.)

$$\forall x \exists y\,\big[\big(x \ge 0 \wedge x \le y\big) \ \Rightarrow\ \forall n\,\big(T(y,n) \vee \neg S(x,n)\big)\big]$$

$$\forall x \exists y\,\big[\big(x \ge 0 \wedge x \le y\big) \wedge \neg\forall n\,\big(T(y,n) \vee \neg S(x,n)\big)\big]$$

$$\forall x \exists y\,\big[\big(x \ge 0 \wedge x \le y\big) \wedge \exists n \neg\big(T(y,n) \vee \neg S(x,n)\big)\big]$$

$$\forall x \exists y\,\big[\big(x \ge 0 \wedge x \le y\big) \wedge \exists n\big(\neg T(y,n) \wedge S(x,n)\big)\big]$$

4. *(15 pts.)* State *Pasch's theorem* and draw a sketch to illustrate it.

[See p. 114.]

5. *(20 pts.)* Do **ONE** of these [(A) or (B)]. *(Extra credit for doing both is limited to 10 points.)*

(A) Prove *Proposition 3.12*: If AC $\cong$ DF, then for any point B between A and C, there is a unique point E between D and F such that AB $\cong$ DE.

[See p. 124.]

(B) State and prove the *crossbar theorem*. For this purpose you can make use of (**DON'T** reprove) *Proposition 3.8*: If D is in the interior of $\angle$ CAB, then (a) so is every other point on ray $\overrightarrow{\mathrm{AD}}$ except A; (b) no point on the opposite ray to $\overrightarrow{\mathrm{AD}}$ is in the interior of $\angle$ CAB; (c) if $\mathrm{C} * \mathrm{A} * \mathrm{E}$, then B is in the interior of $\angle$ DAE.

[See p. 116.]

6. *(Essay – 20 pts.)* *IMPROVEMENTS IN THE REWRITE GET ONLY HALF CREDIT. E.G., IF YOUR INITIAL SCORE IS 16, YOUR FINAL SCORE WON'T EXCEED 18.*

Explain what the *Euclidean parallelism property* is, and what it means to say that the Euclidean parallel postulate is *independent* of other axioms. Stress the importance of *models*, and illustrate these ideas in the context of finite incidence geometries (sets of finitely many points and lines that satisfy the three "I" axioms).