Math. 467, Sec. 500 Spring, 2011

Modern Geometry

Course Content: Euclidean geometry is developed from a modern viewpoint, following the axioms proposed by Hilbert, with special attention to the question of which conclusions are independent of the parallel postulate. Then we examine a particular non-Euclidean geometry, the hyperbolic one. The historical and philosophical context of these developments is given considerable attention.

Course objectives:

1. To acquaint students with non-Euclidean geometry, a development of the utmost historical and philosophical significance in the development of mathematics.
2. To help develop facility with logical thinking. (This includes, in particular, the distinction between reasoning from abstract axioms containing undefined terms and reasoning from facts about things we already know something about.)
3. To prepare future teachers of high-school geometry to approach Euclidean geometry from a well-informed, modern perspective. (Note: *Not* "to teach how to teach high-school geometry".)

Prerequisites: Linear algebra (Math. 304 or 222 or equivalent). Some experience with reading and writing proofs.

Classes: TR 2:20–3:35, BLOC 164

Web page: `http://calclab.math.tamu.edu/~fulling/m467/s11/`

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If I am not in my office, you can leave a note in my mailbox (in the room opposite the math department office, 6th floor of Blocker) or in the wall pouch beside my office door.

Tentative office hours: M 3:00–3:50, W 2:00-2:50, R 11:00–11:50
Permanent office hours will be announced later.

Textbook: M. J. Greenberg, *Euclidean and Non-Euclidean Geometries: Development and History*, 4th ed., Freeman, 2007. The course covers the first 7 or 8 chapters of the book.

Grading system:

Midterm test:	100
Final exam:	150
Homework:	100
Papers:	$50 \times 2 = 100$
Class participation:	50
Total	500

The "curve" will be at least as generous as the "standard" scale [i.e., 90% (= 450 pts) will guarantee an **A**, etc.].

Due dates of special papers: Tuesday, March 1; Tuesday, April 26.

Date of midterm test: Thursday, March 3.

Final exam: Wednesday, May 11, 1:00–3:00 p.m. It may have a take-home component due earlier.

Papers: Choose a topic from the "Major Exercises" and "Projects" lists in the textbook (or propose a topic of your own). The second one should come from fairly late in the book. (More detailed guidelines will be provided later.) Write carefully and formally ("English counts!").

Class participation: We will sometimes discuss homework problems and other examples at the blackboard (or projector) in class. Sometimes I'll assign problems for you to work on in class in groups. At other times volunteers and random draftees will simply be called on. (You may also be called to the board to help me introduce a new concept or technique "Socratically". In such cases a good participation score is attained merely by being alert and cooperative.)

Make-up tests: Make-up tests are very hard to grade fairly, and they absorb a large amount of my time which would be better spent for the benefit of the whole class. Please cooperate in making these incidents as rare as possible. If you miss (or foresee that you will miss) a test, it is *your* responsibility to contact me as soon as possible to request, justify, and schedule a make-up test. (If you can't reach me directly, you can leave a message at the Math Department office, (979) 845–3261.) If the absence is not clearly excused under the Attendance section of *Student Rules*, the request may be denied.

An Aggie does not lie, cheat, or steal or tolerate those who do. See Honor Council Rules and Procedures, `http://www.tamu.edu/aggiehonor` .

Plagiarism: Finding information in books or on the Internet is praiseworthy; *lying* (even by silence) about where it came from is academic dishonesty. Whenever you copy from, or "find the answer" in, some other source, *give a footnote or reference*. Otherwise, you are certifying that it is your own work.

Joint work: On a homework assignment (*not* a take-home test!) discussion with other students is permitted, even encouraged. However, the grader will not give homework credit for "work" that is parasitical (and your test scores will suffer, too!). To forestall problems, please follow these policies: (1) When two or more students work together on an assignment, they should all indicate so on their papers. (2) If the cooperation is of the divide-and-conquer variety, you are certifying that you *have studied and understand* every problem solution on your paper. Mindless copying is dishonest and academically worthless.

Copyright: Course materials (on paper or the Web) should be assumed to be copyrighted by the instructor who wrote them or by the University.

Disabilities: The Americans with Disabilities Act (ADA) is a federal anti-discrimination statute that provides comprehensive civil rights protection for persons with disabilities. Among other things, this legislation requires that all students with disabilities be guaranteed a learning environment that provides for reasonable accommodation of their disabilities. If you believe you have a disability requiring an accommodation, please contact the Disability Services Office in Cain Hall, Room B118, or call 845–1637.

Class schedule (approximate)

Week 1: Review of traditional Euclidean axiomatic geometry

Week 2: Logic, verbal and symbolic: Quantifiers, propositional connectives, survey of techniques

Week 3: Consistency and models, incidence axioms, quick tour of projective and affine spaces

Week 4: Betweenness axioms

Week 5: Betweenness axioms (student team exercises)

Week 6: Congruence axioms

Week 7: Midterm test, continuity axioms, parallelism axiom

Week 8: Neutral geometry (without a parallelism axiom), propositions equivalent to parallelism or its negation

Week 9: Student team exercises on material of Week 8

Week 10: Saccheri and Lambert quadrilaterals, early modern history of parallelism, equivalent postulates

Week 11: Discovery of non-Euclidean geometry, hyperbolic parallelism and limiting parallel rays, inconsistency of elliptic parallelism with the Hilbert axioms

Week 12: Arc length, hyperboloidal model of hyperbolic geometry, implications of consistency of hyperbolic geometry

Week 13: Klein and Poincare models of hyperbolic geometry

Week 14: Negative and positive curvature, Beltrami pseudosphere, difficulty of axiomatizing elliptic geometry