Math. 467 (Fulling) 3 March 2011

Midterm Test – Solutions

Name: ______________________________

1. *(Multiple choice – each 5 pts.) (Circle the correct capital letter.)*

   (a) Which of these statements is **false**? A right angle

   (A) is congruent to its supplement.
   (B) has angle measure 90°.
   (C) is defined as the space between two perpendicular rays.
   (D) is congruent to every other right angle.
   (E) can be constructed by straightedge and compass at any given point on a given line.

C

   (b) In the simplest incidence geometries, the lines are the sets containing exactly 2 points. The smallest geometry of this type that displays the *hyperbolic* parallelism property has

   (A) 2 points.
   (B) 3 points.
   (C) 4 points.
   (D) 5 points.
   (E) 6 points.

D

   (c) Axiom C-4 authorizes us to construct an angle $\angle \mathrm{B'A'C'}$ congruent to a given angle $\angle \mathrm{BAC}$ adjacent to a given ray $\overrightarrow{\mathrm{A'B'}}$. The result is unique provided that

   (A) the *side* of the line $\overleftrightarrow{\mathrm{A'B'}}$ on which the new angle appears is prescribed.
   (B) segment AB is congruent to segment A′B′.
   (C) the Euclidean parallel postulate holds.
   (D) $\angle \mathrm{BAC}$ is a right angle.
   (E) congruence of angles is an equivalence relation.

A

   (d) Which of these is **true**?

   (A) The crossbar theorem implies that every point in the interior of an angle lies on a segment joining a point on one side of the angle to a point on the other side.
   (B) Pasch's theorem is roughly equivalent to Euclid's 4th postulate.
   (C) Pappus's short proof that an isosceles triangle has equal angles is invalid, because flipping a triangle through the third dimension goes beyond the reasoning allowed by the Hilbert axioms.
   (D) Unlike segment addition, angle addition is *not* a Hilbert axiom, because it can be proved from the SAS axiom.
   (E) If D is in the interior of $\angle \mathrm{CAB}$, then so is every other point on ray $\overrightarrow{\mathrm{DA}}$ except A.

D

(e) The exercises in Chapter 1 showed how to define all of the following **except**

(A) vertical angles.
(B) the length of a segment.
(C) the midpoint of a segment.
(D) the bisector of an angle.
(E) the perpendicular bisector of a segment.

B

2. *(15 pts.)* Express in logical notation (quantifiers and connectives, plus a bare minimum of English):

   Taxpayer Smith may claim his daughter as a dependent, provided that she is not married and filing a joint return, if he provides more than half of her support, or if he and another person together provide more than half of her support and he paid over 10% of her support.

   (Let S be the *name* Smith, so that it is not necessary to put S inside a quantifier. Let $P(x, d, z)$ mean that x pays more than z percent of d's support.)

Oops, I forgot to define a notation for "x and y together pay more than z percent of d's support." Many students improvised something such as $P(x + y, d, z)$ or $P(x \cup y, d, z)$. I'll use the latter.

There are many acceptable answers, since the rules of how much to put into symbols are not clear. I was hoping for something like this:

$$\begin{aligned}&\forall d\,\{d \text{ is daughter of } S \wedge d \text{ is not married and filing a joint return}\\ &\quad \wedge [P(S, d, 50) \vee \exists y(P(S \cup y, d, 50) \wedge P(S, d, 10))]\\ &\quad \Rightarrow\ S \text{ may claim } d \text{ as a dependent}\}\end{aligned}$$

3. *(15 pts.)* State the three *betweenness* axioms that do not involve the concept "side".

[See B-1, B-2, B-3 in Chapter 3.]

4. *(20 pts.)* Prove **ONE** of these: *(At most 10 points extra credit for both.)*

   (A) The SSS triangle congruence theorem (which you should state).

   (B) Proposition 3.3(b): If A ∗ B ∗ C and A ∗ C ∗ D, then A ∗ B ∗ D.

To give a self-contained proof of 3.3(b) you have to reprove 3.3(a) first; see pp. 112–113 and the team paper on the Fall 2009 web page. (I didn't remember that when I chose this problem for the test.)

Name: ____________________

5. *(Essay – 25 pts.)* Let $\mathcal{S}$ be a set. Recall that an *equivalence relation* on $\mathcal{S}$ is a binary relation $\sim$ with the properties (for all A, B, C in $\mathcal{S}$)
   (1) reflexivity: $A \sim A$,
   (2) symmetry: $A \sim B \;\Rightarrow\; B \sim A$,
   (3) transitivity: $A \sim B \wedge B \sim C \;\Rightarrow\; A \sim C$.
   Recall also that a *partition* of $\mathcal{S}$ is a collection $\{S_i\}$ of subsets of S with the properties
   (4) disjointness: $S_i \cap S_j = \emptyset$ if $i \neq j$,
   (5) exhaustiveness: $S = \bigcup_i S_i$.

   Show how every equivalence relation on $\mathcal{S}$ determines a partition of $\mathcal{S}$, and how every partition of $\mathcal{S}$ determines an equivalence relation on $\mathcal{S}$. (In each of the two cases, explain how the new entity is defined from the old one and prove that it satisfies all the clauses in the definition.)