Group Eta

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Proposition 3.1 (ii) $\overrightarrow{AB} \cup \overrightarrow{BA} = \left\{\overleftrightarrow{AB}\right\}$

Proof. We want to prove that $\overrightarrow{AB} \cup \overrightarrow{BA} = \left\{\overleftrightarrow{AB}\right\}$.

Case 1: Show $\overrightarrow{AB} \cup \overrightarrow{BA} \subset \left\{\overleftrightarrow{AB}\right\}$.We know by definition of a ray that $\overrightarrow{AB} \subset \left\{\overleftrightarrow{AB}\right\}$, and $\overrightarrow{BA} \subset \left\{\overleftrightarrow{AB}\right\}$, and therefore, $\overrightarrow{AB} \cup \overrightarrow{BA} \cup \left\{\overleftrightarrow{AB}\right\}$.

Case 2: We want to show that $\overrightarrow{AB} \in \overrightarrow{AB} \cup \overrightarrow{AB}$

i Let $P \subset \overleftrightarrow{AB}$, and let P be the endpoints, so $P = A$ and $P = B$. Then $P \subset \overrightarrow{AB}$ and $P \subset \overrightarrow{BA}$ by definition of a ray, so $P \subset \overrightarrow{AB} \cup \overrightarrow{BA}$.

ii Let $A * P * B$, so P is between A and B. By B-1, $P \cup \overrightarrow{AB}$.

iii If $P * A * B$, then $P \in \overrightarrow{BA}$ by the definition of a ray.

iv If $P * B * A, P \in \overrightarrow{AB}$ by the definition of a ray.

□