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Group Question 1

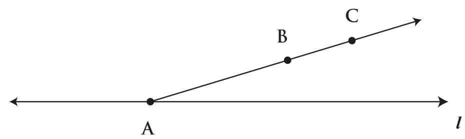

Figure 1

Lemma 1 Given a line l, a point A on l, and a point B not on l. Then every point of the ray $\overrightarrow{AB}$ (except A) is on the same side of l as B.

Proof. Choose a point C on $\overrightarrow{AB}$ (except A). Then by construction, BC does not intersect l and hence all points of the ray $\overrightarrow{AB}$ (except A) are on the same side of l as B. □

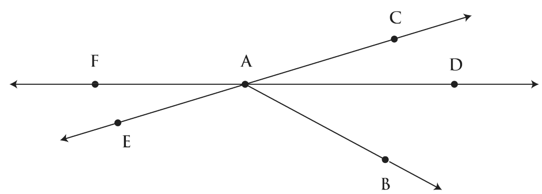

Figure 2

Proposition 3.8 If D is in the interior of $\angle CAB$, then: (a) so is every other point on ray $\overrightarrow{AD}$ except A; (b) no point on the opposite ray to $\overrightarrow{AD}$ is in the interior of $\angle CAB$; and (c) if $C * A * E$, then B is in the interior of $\angle DAE$ (see figure 1)

Proof.

a. Since D is an interior point of $\angle CAB$ by hypothesis, it lies on the same side of $\overleftrightarrow{AB}$ as C. By Lemma 1, this means every point of $\overrightarrow{AD}$ (except A) lies on the same side of $\overleftrightarrow{AB}$ as C. An analogous argument shows that every point of $\overrightarrow{AD}$ (except A) lies on the same side of $\overleftrightarrow{AC}$ as B, hence every point of $\overrightarrow{AD}$ (except A) lies in the interior of $\angle CAB$.

b. Denote a point F on the ray opposite to $\overrightarrow{AD}$. Then D and F are on opposite sides of $\overleftrightarrow{AB}$ by construction. Moreover, since D is an interior point of $\angle CAB$ by hypothesis, it lies on the same side of $\overleftrightarrow{AB}$ as C. By Betweenness Axiom 4–iii, it follows that F and C are on opposite sides of $\overleftrightarrow{AB}$ and hence F cannot be an interior point of $\angle CAB$.

c. The points C and B lie on opposite sides of $\overleftrightarrow{AD}$ by part a. Since E and C are on opposite sides of $\overleftrightarrow{AD}$ by construction, it follows that E and B lie on the same side of $\overleftrightarrow{AD}$ by Betweenness Axiom 4–ii. By Lemma 1, this means that every point of $\overrightarrow{AB}$ (except A) lies on the same side of $\overleftrightarrow{AD}$ as E. Since D and B lie on the same side of $\overleftrightarrow{AC}$ by hypothesis, and since $\overleftrightarrow{AE}$ coincides with $\overleftrightarrow{AC}$, it follows that D and B lie on the same side of $\overleftrightarrow{AE}$. By Lemma 1, this means that every point of $\overrightarrow{AB}$ (except A) lies on the same side of $\overleftrightarrow{AE}$ as D. From here we can conclude that B lies in the interior of $\angle DAE$.

□