**Proposition 4.10**

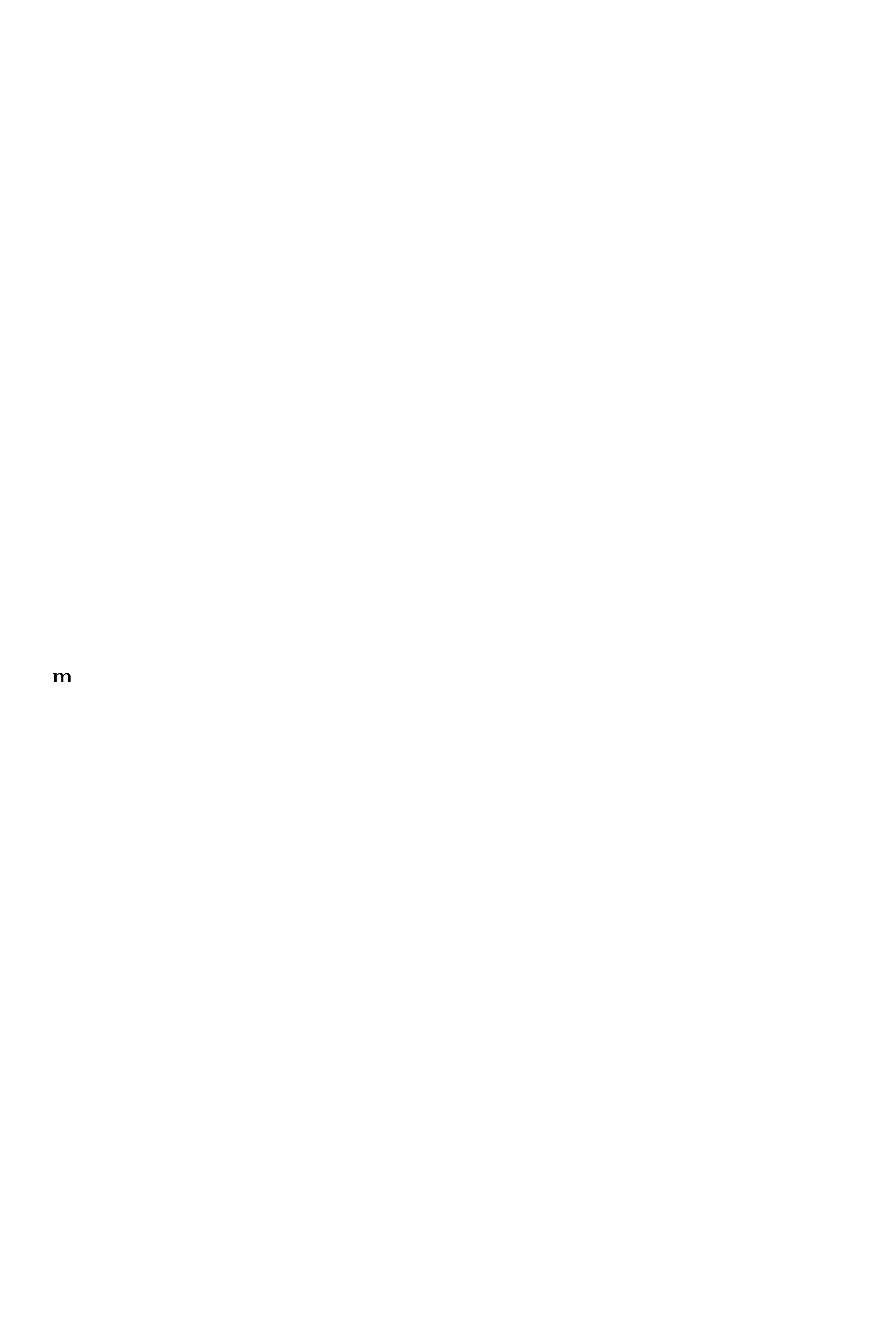
Proposition 4.10: Hilbert’s Euclidean parallel postulate if and only if; if k is parallel to l, m is perpendicular to k, and n is perpendicular to l, then either m=n or m is parallel to n.

**Proof**

Assume the Hilbert’s Euclidean parallel postulate is true, and k is parallel to l, m is perpendicular to k, and n is perpendicular to l. Let m≠n, by proposition 4.9, m has to be perpendicular to l, and by corollary 1 (pg.163), m is parallel to n

k

l

n

m

Assume that whenever *k* is parallel to *l*, *m* is perpendicular to *k*, and *n* is perpendicular *l*, it follows that either *m* is equal to *n* or *m* is parallel to *n*.

We need to show Hilbert’s Euclidean parallel postulate. So assume that *l* is some line, p is a point not on *l*, and we need to show there is at most one line through P parallel to *l*.

We know that there is at least one line m through P parallel to *l* by Corollary 2 (If l is any line and P is any point not on *l*, there exists at least one line *m* through P parallel to *l*). Suppose that *n* is a line through P parallel to *l*: We must show *m=n*. Let *t* be the perpendicular to *l* through P, with foot Q on *l*.

Let *s* be the perpendicular to *n* through P. Since *t* is perpendicular to *l*, at Q, and *s* is perpendicular to *n*, at P, and *l* is perpendicular to *n*, by our assumption either *s=t* or *s* is parallel to *t*. But P is on both *t* and *s*, so they cannot be parallel, so *s=t*. But the ray through P perpendicular to *t* is unique, and both *m* and *n* go through P perpendicular to *t*, so *m=n*.