Team Beta
Texas A&M University - Math 467
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Proposition 4.5

To Show: In $\triangle ABC$, $AB > BC$ iff $\angle BCA > \angle CAB$.

Proof. $(\Rightarrow)$: Suppose $AB > BC$. Pick D with $A * D * B$ and $BD \cong BC$. Since B is the unique point of intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$, D is not incident to $\overleftrightarrow{BC}$. Hence, BDC is a triangle. Similarly, ADC is a triangle.

Since $BD \cong BC$, $\triangle BDC$ is isosceles with $\angle BDC \cong \angle BCD$. By the exterior angle theorem applied to $(\triangle ADC, \angle BDC)$, we get $\angle CAB < \angle BDC$. Since $A*D*B$, we have $\angle BCD < \angle BCA$ by the definition of angle ordering. Putting it all together, $\angle CAB < \angle BCD \cong \angle BCD < \angle BCA$.

$(\Leftarrow)$: Suppose $\neg(AB > BC)$. Then either $AB = BC$ or $AB < BC$. In the former case, $\triangle ABC$ is isosceles with $\angle BCA = \angle CAB$. In the latter case, apply $(\Rightarrow)$ to $\triangle CBA$ to get $\angle A > \angle C$. $\square$